1.

Použijeme substituci $x = \operatorname{tg} t,\ t \in \left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right),\ \mathrm{d}x = \dfrac{\mathrm{d}t}{\cos^2 t}\,;\ \ t = \operatorname{arctg} x.$

Platí

$\sqrt{1+x^2} = \sqrt{1+\operatorname{tg}^2 t} = \sqrt{\dfrac{1}{\cos^2 t}} = \dfrac{1}{|\cos t|} = \dfrac{1}{\cos t}\,,$ neboť pro $t \in \left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ je $\cos t > 0.$

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x^2)^5}} = \int \frac{1/\cos^2 t}{1/\cos^5 t}\ \mathrm{d}t = \int \cos^3 t\ \mathrm{d}t = \int \left(1-\sin^2 t\right)\cos t\ \mathrm{d}t$$

Nyní použijeme substituci $u = \sin t,\ u \in (-1,1),\ \mathrm{d}u = \cos t\ \mathrm{d}t.$

$$\int \left(1-\sin^2 t\right)\cos t\ \mathrm{d}t = \int \left(1-u^2\right)\ \mathrm{d}u = u - \tfrac{1}{3}u^3 = \sin t - \tfrac{1}{3}\sin^3 t = \sin\operatorname{arctg} x - \tfrac{1}{3}\sin^3\operatorname{arctg} x$$

Nalezenou primitivní funkci můžeme popř. upravit. Platí

$$\sin\operatorname{arctg} x = \sin t = \frac{\operatorname{tg} t}{\sqrt{1+\operatorname{tg}^2 t}} = \frac{x}{\sqrt{1+x^2}}\,,$$

neboť pro $t \in \left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ mají $\sin t$ a $\operatorname{tg} t$ stejná znaménka a $\operatorname{tg} t = \operatorname{tg}\operatorname{arctg} x = x$. Proto

$$\sin\operatorname{arctg} x - \tfrac{1}{3}\sin^3\operatorname{arctg} x = \frac{x}{\sqrt{1+x^2}} - \frac{1}{3}\cdot\frac{x^3}{\sqrt{(1+x^2)^3}} = \frac{2x^3+3x}{3\sqrt{(1+x^2)^3}}\,.$$

Závěr:

$$\int \frac{\mathrm{d}x}{\sqrt{(1+x^2)^5}} = \sin\operatorname{arctg} x - \tfrac{1}{3}\sin^3\operatorname{arctg} x = \frac{2x^3+3x}{3\sqrt{(1+x^2)^3}}\,,\quad x \in \mathbf{R}$$