# 4. CVIČENÍ

ALEŠ NEKVINDA

## Příklady na Fubiniho větu ve dvojném integrálu

$$\iint_M (2x+y)dxdy, \qquad M = \{[x,y]; x \geq 0, y \geq 0, x+y \leq 3\}$$
$$\iint_M xydxdy, \qquad M = \{[x,y]; x \geq 0, y \geq 0, \sqrt{x}+\sqrt{y} \leq 1\}$$
$$\iint_M xy^2dxdy, \qquad M = \{[x,y]; x \geq 0, x^2+4y^2 \leq 1\}$$
$$\iint_M y^2\sqrt{1-x^2}dxdy, \qquad M = \{[x,y]; y \geq 0, x^2+y^2 \leq 1\}$$
$$\iint_M \frac{2x}{x^2+y^2}dxdy, \qquad M = \{[x,y]; x \geq 0, y \leq 1, \sqrt{x} \leq y\}$$
$$\iint_M xydxdy, \qquad M = \{[x,y]; x \geq 0, -x \leq y \leq 2x-x^2\}$$

## Příklad

Spočtěte

$$\iint_M \frac{2x}{x^2+y^2}dxdy,$$

kde

$$M = \{[x,y]; x \geq 0, y \leq 1, \sqrt{x} \leq y\}.$$

1.postup:

$$\iint_M \frac{2x}{x^2+y^2}dxdy = \int_0^1 \int_0^{y^2} \frac{2x}{x^2+y^2}dxdy = \int_0^1 [\ln(x^2+y^2)]_0^{y^2} dy$$
$$= \int_0^1 (\ln(y^4+y^2) - \ln y^2)dy = \int_0^1 \ln(y^2+1)dy$$
$$= [y\ln(y^2+1)]_0^1 - \int_0^1 y\frac{2y}{y^2+1}dy = \ln 2 - 2\int_0^1 \frac{y^2}{y^2+1}dy$$
$$= \ln 2 - 2\int_0^1 \Big(1 - \frac{1}{y^2+1}\Big)dy = \ln 2 - 2[y - \operatorname{arctg} y]_0^1$$
$$= \ln 2 - 2(1-\pi/4) = \ln 2 + \pi/2 - 2.$$

2.postup:

$$\iint_M \frac{2x}{x^2+y^2}dxdy = \int_0^1 \int_{\sqrt{x}}^1 \frac{2x}{x^2+y^2}dydx = \int_0^1 \Big[2x\frac{1}{x}\operatorname{arctg}\frac{y}{x}\Big]_{\sqrt{x}}^1 dx$$

$$= 2\int_0^1 \Big(\operatorname{arctg}\frac{1}{x} - \operatorname{arctg}\frac{1}{\sqrt{x}}\Big)dx$$

$$= 2\Big[x\Big(\operatorname{arctg}\frac{1}{x} - \operatorname{arctg}\frac{1}{\sqrt{x}}\Big)\Big]_0^1 - 2\int_0^1 x\Big(\frac{1}{1+\frac{1}{x^2}}\Big(-\frac{1}{x^2}\Big) - \frac{1}{1+\frac{1}{x}}\Big(-\frac{1}{2}x^{-3/2}\Big)\Big)dx$$

$$= 2\int_0^1 \Big(\frac{x}{1+x^2} - \frac{1}{2}\frac{\sqrt{x}}{1+x}\Big) = [\ln(1+x^2)]_0^1 - \int_0^1 \frac{2t^2}{1+t^2}dt$$

$$= \ln 2 - 2\int_0^1 \Big(1 - \frac{1}{1+t^2}\Big)dt = \ln 2 - 2[t - \operatorname{arctg} t]_0^1$$

$$= \ln 2 - 2(1-\pi/4) = \ln 2 + \pi/2 - 2.$$

<u>Domácí cvičení</u>

(1)

$$\iint_M (x^2+y^2)dxdy,\ M = \{[x,y]; |x|+|y| \le 1\} \qquad \frac{2}{3}$$

(2)

$$\iint_M (x+y^2)dxdy,\ M = \{[x,y]; x^2 \le y \le \sqrt{x}\} \qquad \frac{33}{140}$$

(3)

$$\iint_M \frac{x^2}{y^2}dxdy,\ M = \{[x,y]; 1 \le x \le 2, 1 \le xy, y \le x\} \qquad \frac{9}{4}$$

(4)

$$\iint_M x^2 \mathrm{e}^{-y}dxdy,\ M = \{[x,y]; 0 \le x \le 2, 0 \le y \le x^3\} \qquad \frac{1}{3}(7+\mathrm{e}^{-8})$$

(5)

$$\iint_M \sqrt{4x^2-y^2}dxdy,\ M \text{ je trojúhelník s vrcholy } [0,0],[1,0],[1,1] \qquad \frac{3\sqrt{3}+2\pi}{72}$$