Katedra matematiky
Fakulta stavební ČVUT v Praze

SBORNÍK

abstraktů

Studentské konference
a
Rektorysovy soutěže

8. listopadu 2024
Praha

Vydavatel: Katedra matematiky, FSv ČVUT v Praze
Praha 2024
Editor: Stanislav Olivík

Podpořeno grantem SGS ČVUT, SVK 01/24/F1

Předmluva

Katedra matematiky Fakulty stavební ČVUT tradičně pořádá Studentskou vědeckou konferenci, v jejímž rámci se koná Rektorysova soutěž. Připomeňme si, že v roce 2007 se konal 0. ročník, letos se koná již 17. ročník. O organizaci jsme se v minulých letech dělili s Katedrou matematiky Fakulty jaderné a fyzikálně inženýrské ČVUT v Praze.

Letošní ročník se stejně jako v loňském roce koná formou videokonference.

Prezentované práce jsou z různých oblastí aplikované matematiky. Mnoho z nich souvisí s bakalářskými, diplomovými nebo disertačními pracemi účastníků konference.

Organizátoři konference by zde rádi poděkovali především účastníkům konference a Rektorysovy soutěže, dále vedoucím studentských prací, oponentům a členům odborné hodnotící poroty za jejich práci, která přispěla k vysoké úrovni a prestiži konference. Je také potřeba zmínit přínos správce webu Studentské konference a Rektorysovy soutěže Ing. Stanislava Olivíka, jehož všestanná činnost po mnoho let napomáhá hladkému průběhu akce.

Studentská vědecká konference a Rektorysova soutěž je částečně financována z grantu SVK 01/24/F1 uděleného naší katedře grantovou komisí Studentské grantové soutěže ČVUT. Tímto děkujeme grantové komisi za podporu.

Informace o Studentské vědecké konferenci a Rektorysově soutěži lze nalézt na webových stránkách https://mat.fsv.cvut.cz/rektorys/soutez/ .

V Praze dne 7. 11. 2024

Doc. Jozef Bobok
Dr. Martin Soukenka

Karel Rektorys

Prof. RNDr. Karel Rektorys, DrSc. (1923-2004) působil na ČVUT od roku 1954 do roku 2004, tedy celých 50 let. Stal se významnou osobností mezi vědci. Proslavil se zejména metodou časové diskretizace při řešení parciálních diferenciálních rovnic. Profesor Rektorys měl obrovskou autoritu i jako pedagog. Jeho přednášky se staly fenoménem. Jako vystudovaný matematik dokázal překlenout hranice matematiky a inženýrských oborů. Podílel se například na projektu stavby Orlické přehrady. Byl autorem řady publikací, Variační metody v inženýrských problémech a v problémech matematické fyziky, Metoda časové diskretizace a parciální diferenciální rovnice, Co je a k čemu je vyšší matematika, a byl vedoucím kolektivu autorů světoznámého Přehledu užité matematiky.

Obsah

<u>Soutěžní příspěvky</u>

Soutěžní příspěvky

Experimental and numerical modelling of PVB foil

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Abstract. Laminated glass is a composite material made of glass plates and polymer interlayers which ensure binding of the plates and have an important safety function. The most commonly used material for the interlayer is polyvinyl butyral (PVB), a polymer with significantly time and temperature dependent properties. Due to its time dependent behaviour the material can be described as viscoelastic. To describe a viscoelastic material the models consisting of elastic springs and viscous dashpots are used. The description can be also provided using the fractional viscoelasticity which bases on the theory of derivatives and integrals of non-integer order. Fractional viscoelasticity introduces another rheological element, a springpot. This element behaves as viscoelastic itself and its connection with more elastic or viscous elements opens the door to other, more advanced, models to describe viscoelastic materials. This thesis mainly focuses on the generalized Maxwell model in its standard as well as its fractional form. Behaviour of these two models is examined during the numerical analysis using the finite difference method for several loading cases and for varying model parameters. To describe the response of a real material the parameters of the models need to be calibrated. To this end, an extensive experimental program was executed. The viscoelastic behaviour of PVB was examined using the dynamic shear rheometer. The acquired measurements were then adopted in the optimization process of material parameters of both models to provide predictions which match the response of PVB observed experimentally.

Acknowledgements. This publication was supported by the Czech Science Foundation, the grant No. 22-15553S and by the Grant Agency of the Czech Technical University in Prague, grant No. SGS24/038/OHK1/1T/11.

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Mathematical modeling of phylogenetic compression

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Abstract. Comprehensive genome collections are growing exponentially, outpacing computational capacities and rendering their storage and analysis increasingly challenging. While recent advancements in highly optimized alignment-based and k-mer approaches have provided various one-time improvements, they overall struggle to address the underlying scalability challenge. A recent approach called phylogenetic compression leverages evolutionary history to enhance algorithms and data structures, resulting in an improvement of one to two orders of magnitude over state-of-the-art methods for compression and search of large and diverse bacterial genome collections. However, despite the clear performance improvement with phylogenetic compression, its theoretical foundations are yet to be established. Here, we introduce a formal framework to study the mathematical principles of phylogenetic compression. We model it with binary matrices and run-length encoding, and formalize compression as an optimization problem. We demonstrate that while this problem is generally NP-hard for agnostic data, phylogenetic compression can achieve optimal solutions in polynomial time under simplified evolutionary models, aligning with observed practical improvements. These findings not only provide a theoretical foundation for phylogenetic compression modeling but also open the way for future improvements and innovations of scalable tools for storage and analysis of large genomic datasets, addressing one of the key challenges in computational biology today.

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Optimising truss-based metamaterials for unimodal and pentamodal behaviour

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Abstract. Our study focuses on the optimisation of the internal structure of unimodal and pentamodal metamaterials, modelled as three-dimensional linear elastic lattice structures. For optimisation, we represent the metamaterials with discrete truss models of their respective Periodic Unit Cells (PUCs), whose effective response is determined by the first-order numerical homogenisation. The optimisation is formulated as an inverse homogenisation problem with objective functions comprising a ratio of selected eigenvalues of the effective stiffness matrix, which allows us to dispense with the traditional volume constraint and solve the optimisation problem with a simple gradient method combined with the line search method. We demonstrate the efficacy of the formulation with a design of a unimodal material compliant in a chosen shear deformation mode and we also show that our formulation recovers the traditional pentamodal metafluid.

Acknowledgements. This work was supported by the Czech Science Foundation, Project No. GA22-15524S.

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Optimální diskrétní Hardyho nerovnosti vyššího řádu

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Abstrakt. V této práci nejprve shrneme historický vývoj a známé důkazy klasické diskrétní Hardyho nerovnosti. Dále prozkoumáme podobnosti a rozdíly mezi její diskrétní a spojitou verzí, načež se zaměříme na vylepšenou Hardyho nerovnost pro diskrétní Laplaceův operátor Δ objevenou roku 2018 autory Keller-Pinchover-Pogorzelski. Hlavním výsledkem je pak nalezení obdobných optimálních nerovností pro libovolnou přirozenou mocninu ℓ Laplaceova operátoru. Pro $\ell = 2$ jsme objevili optimální Rellichovu nerovnost, vylepšujíce tak dosavadní nejlepší váhy od autorů Gerhat-Krejčiřík-Štampach a Huang–Ye. Pro $\ell \geq 3$ jsme dokázali hypotézu od Gerhat-Krejčiřík-Štampach a vylepšili klasické Birmanovy váhy objevené autory Huang-Ye na optimální.

Editor	Stanislav Olivík
Název díla	Sborník abstraktů Studentské konference a Rektorysovy soutěže
Vydalo	České vysoké učení technické v Praze
Zpracovala	Fakulta stavební
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Počet stran	20
Vydání	1.

Neprodejné.

Praha 2024