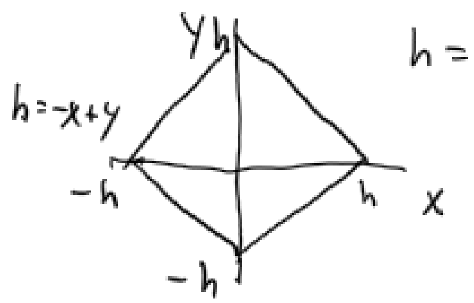
① MA02 Př.
11. 11. '20

Př. (vrstevnice) : Funkce $f(x,y) = |x| + |y|$.

Def. obor $D_f = \mathbb{R}^2$ (celá rovina), $H_f = [0, +\infty)$

Vrstevnice : $h < 0$ vrstevnice $V_h = \emptyset$ , $h = 0 \Rightarrow V_h = \{(0,0)\}$

$h > 0$




$V_h$ je hranice čtverce daného body $(h,0), (0,h)$,
vrcholy $(-h,0), (0,-h)$

Vrstevnicová plocha : Funkce $f(x,y,z) = \dfrac{x^2+y^2}{z}$

$h = 0$ osa z bez bodu $(0,0,0)$. $h \neq 0$ $h = \dfrac{x^2+y^2}{z} \Rightarrow z = \dfrac{1}{h}(x^2+y^2)$
bez bodu $(0,0,0)$

vrstevnicová plocha $\rightarrow$ rotační paraboloid

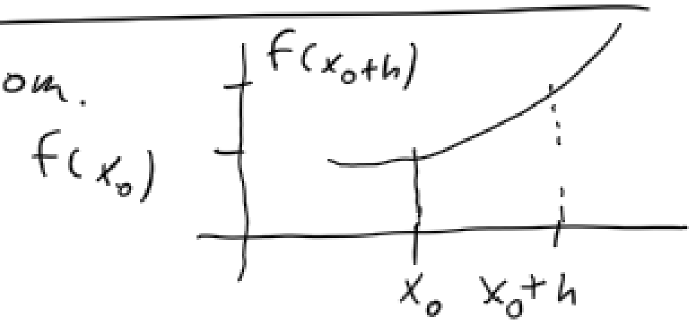
Operace s fcemi více prom.

Př. $f(x,y) = x^2 \ln y$ , $g(x,y) = e^x y^3$

$(f+g)(x,y) = x^2 \ln y + e^x y^3$

Derivace fce 1 prom.

$$\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$




Derivování

Př. $f(x,y) = \sin(x^2 - y^3)$ , $D_f = \mathbb{R}^2$

$$\frac{\partial f}{\partial x}(x,y) = \cos(x^2 + y^3) \cdot 2x \qquad \frac{\partial f}{\partial y}(x,y) = \cos(x^2 - y^3)(-3y^2) = -3y^2 \cos(x^2 - y^3)$$

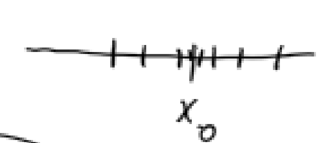
$$\frac{\partial^2 f}{\partial x^2}(x,y) = 2\cos(x^2 - y^3) + 2x(-\sin(x^2 - y^3) \cdot 2x) = 2\cos(x^2 - y^3) - 4x^2 \sin(x^2 - y^3)$$

② MA02 Př. 11.11.2020 $\frac{\partial^2 f}{\partial x \partial y}(x,y) = -2x\sin(x^2-y^3)(-3y^2) = 6xy^2\sin(x^2-y^3)$

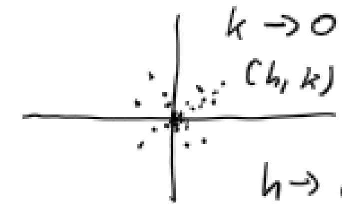
$$\frac{\partial^2 f}{\partial y^2}(x,y) = -6y\cos(x^2-y^3) - 3y^2\left(-\sin(x^2-y^3)\right)(-3y^2) = -6y\cos(x^2-y^3) - 9y^4\sin(x^2-y^3)$$

$$\frac{\partial^2 f}{\partial y \partial x}(x,y) = -3y^2\left(-\sin(x^2-y^3)\right)2x = 6xy^2\sin(x^2-y^3) = \frac{\partial^2 f}{\partial x \partial y}(x,y)$$

Totální diferenciál. Limita 1D




2D




Př. Aplikace - přibližná hodnota funkce

Vypočítejte přibl. hodnotu výrazu $\frac{\sqrt[4]{0,97}}{1,02^3 \sqrt[3]{0,99}}$. Zaveďme funkci $f(x,y,z) = \frac{x^{1/4}}{y^3 z^{1/3}}$

$x_0 = 1,\ y_0 = 1,\ z_0 = 1$

$x_1 = 0,97,\quad y_1 = 1,02 \quad z_1 = 0,99$

$dx = -0,03\ ,\ dy = 0,02\ ,\ dz = -0,01$

$$f(x_1, y_1, z_1) \doteq f(x_0, y_0, z_0) + \frac{\partial f}{\partial x}(x_0, y_0, z_0)\,dx + \frac{\partial f}{\partial y}(x_0, y_0, z_0)\,dy + \frac{\partial f}{\partial z}(x_0, y_0, z_0)\,dz$$

$$\frac{\partial f}{\partial x}(x_0 \ldots) = \frac{\frac{1}{4} x_0^{-\frac{3}{3}}}{y_0^3 z_0^{1/3}}, \quad \frac{\partial f}{\partial y}(x_0, \ldots) = -3\frac{x_0^{1/4}}{y_0^4 z_0^{1/3}}, \quad \frac{\partial f}{\partial z}(x_0, \ldots) = -\frac{1}{3}\frac{x_0^{1/4}}{y_0^3 z_0^{4/3}}$$

$$f(x_1, y_1, z_1) \doteq \frac{1}{1\cdot 1} + \frac{\frac{1}{4}\cdot 1}{1\cdot 1}(-0,03) - 3\frac{1}{1\cdot 1}\cdot 0,02 + \left(-\frac{1}{3}\right)\frac{1}{1\cdot 1}(-0,01) = 1 - \frac{0,03}{4} - 0,06 + \frac{0,01}{3} \doteq 0,936$$

Přesněji (kalkulačkou) $f(x_1, y_1, z_1) \doteq 0,93831$