

782. a) $\frac{9}{2}$; b) $\frac{40}{3}$; c) $\frac{1331}{48}$; d) $\frac{10}{3}\sqrt{6}$; e) $2(\pi - \frac{2}{3})$; f) $6\ln 3 + 2$. 783. a) $\frac{1}{3}a^2$; b) $-10 + \frac{25}{2}(\frac{1}{2}\pi + \arcsin \frac{3}{5})$. 784. a) $\frac{1}{2}(\pi - 2)a^2$; b) $\frac{1}{12}(3\sqrt{3} - \pi)a^2$. 785. a) a^2 ; b) $\frac{63}{128}\pi a^2$.
786. $\frac{7}{128}\pi a^2$; d) $\frac{3}{4}\pi a^2$; e) $\frac{1}{2}\pi a^2$. 787. a) $\frac{1}{6}c^2$; b) $\frac{1}{120}a^5b^5/c_8$; c) $\frac{21}{256}\pi ab(a^2 + b^2)/c^2$; d) $\frac{1}{4}\pi ab(a^2 + b^2)/c^2$. 788. $\frac{3}{8}\pi a^2$. 789. $8\pi^3 a^2$. 790. a) $\frac{4}{3}$; b) $\frac{1}{3}(b - a)\ln(d/c)$; c) $\frac{1}{3}(b^2 - a^2)(d^3 - c^3)$. 791.
792. $\frac{40}{3}$. 793. $\frac{48}{5}\sqrt{6}$. 794. $\frac{3}{2}$. 795. $\frac{5}{6}\pi a^4$. 796. $\frac{1}{6}\pi a^3$. 797. $\frac{1}{6}\pi a^3$. 798. $\frac{8}{3}\sqrt{2}\pi a^4$. 799. $\frac{3}{4}$. 800. $\frac{9}{4}a^4$. 801. 45 . 802. $\frac{88}{105}$. 803. $\frac{256}{21}$. 804. $3e - 8$. 805.
806. $\frac{3}{4}\pi a^4$. 807. $64t$. 808. $\pi(1 - e^{-a^2})$. 809. πa^3 . 810. $\frac{9}{2}(3\pi + 20 - 16\sqrt{2})a^3$. 811. $\frac{2}{9}(3\pi - 4)a^3$. 812. $\frac{1}{16}F_3$. 813. $\frac{3}{2}(3\pi + 8)a^3$. 814. $\frac{3}{4}\pi a^3 - (\sqrt{a^2 - b^2})^3$. 815.
816. $\frac{1}{9}(20 - 3\pi)$. 817. $\frac{1}{6}(3\sqrt{3} - 1)a^2$. 818. $\frac{1}{6}(3\sqrt{10} + \ln(3 + \sqrt{10}))\pi$. 819.
820. $\frac{2}{3}(2\sqrt{2} - 1)\pi ab$. 821. $\frac{1}{6}(3\sqrt{3} - 1)$. 822. $\frac{1}{2}(3\sqrt{2} - 2)\pi$. 823.
824. $\sqrt{2}\pi$. 825. $2a^2$. 826. $\frac{1}{2}\pi a^2$. 827. $\frac{12}{5}a^2$. 828. $(\pi - 2)a^2$. 829. $4a^2 \operatorname{arcsim}(b/a)$.
830. $\frac{16}{3}\pi a^2$. 831. $16R^2$. 832. $\pi R\sqrt{R^2 + H^2}$. 833. $\pi(R_1 + R_2)\sqrt{(R_1^2 - R_2^2) + H^2}$.
834. $\frac{135}{4}$. 835. π . 836. $\frac{45}{8}$. 837. $\frac{32}{9}a^3$. 838. $\frac{3}{4}ab$. 839. $(-2, \frac{17}{5})$. 840.
841. $(\frac{1}{16}\pi a^2, 0)$. 842. $(12 - \frac{\pi}{2})$. 843. $(10 - 3\pi, 10 - 3\pi)$. 844.
845. a) $\left(2(\pi^3 - 24\pi + 48), \frac{1}{4}(3\pi - 8)\right)$; b) $\left(\frac{3}{4}\pi a, -\frac{\pi^2}{2} - 3a\right)$. 846. $\frac{2}{3}H \sin a$. 847. $\frac{3}{4}R \frac{\sin^3 a}{2a}$. 848. $(\frac{5}{6}a, 0)$. 849. $I_x = \frac{32}{15}bc^3$, $I_y = \frac{4}{15}b^3c$. 850. $\frac{5}{4}\pi R^4$. 851.
852. $I_p = \frac{1}{8}(2a - \sin 2a)H^4$, $I_q = \frac{1}{8}(2a + \sin 2a)H^4$. 853. $\frac{5}{2}\pi$. 854. $I_x = \frac{1}{4}(3\pi - 8)a^4$, $I_y = \frac{49}{32}\pi a^4$. 855. $I_p = \frac{1}{16}(4a - \sin 4a)R^4$, $I_q = \frac{1}{8}(12a - 8\sin 2a + \sin 4a)R^4$, $I_x = \frac{21}{12}\pi a^4$, $I_y = \frac{32}{15}\pi a^4$. 856. $I_x = I_y = \frac{1}{16}\ln 2 - \frac{75}{8}$.

C1. 4. Nejčetří aplikace dvojného integrálu

760. a) $G_{xy} = R^2$, h: $u = \frac{1}{5}x + \frac{2}{5}y$, v = $-\frac{1}{5}x + \frac{3}{5}y$, $J_g = 5$; b) $G_{xy} = \{(x, y) \in \mathbb{R}^2 : x \in (0, 1) \wedge 0 > y > x\}$, h: $u = x$, v = y/x , $J_g = u$; c) $G_{xy} = (0, +\infty) \times (0, +\infty)$, h: $u = \frac{x}{\sqrt{v}}$, v = \sqrt{xy} , $J_g = 5u$; f) $G_{xy} = (0, +\infty) \times (0, +\infty)$, h: $u = (x^{2/3} + y^{2/3})^{1/2}$, v = $\operatorname{arctg}((y/x)^{1/2})$, $J_g = su \sin^{-1} v \cos^{-1} u$. 761. 2π .
765. $J_g = \frac{8}{3}\pi$. 766. $\frac{1}{6}(\sqrt{2} + \ln(1 + \sqrt{2}))$. 767. $\frac{1}{4}\pi ab(a^2 + b^2)$. 768. $\frac{1}{2}\pi a^3$. 769. $\frac{21}{256}\pi a^3$. 770. $\frac{1}{192}\pi a^2$. 771. $-\frac{3}{2}(\sqrt{3} - 1)$. 772. $\frac{1}{4}\pi ab(a^2 + b^2)$. 773. $\frac{3}{2}\pi(37\ln 37 - 36)$. 774. 28π . 775. $\frac{80}{81}a^6$.
776. $\frac{15}{16}\sqrt{6}$. 777. $\frac{3}{4}$. 778. $\frac{6}{5}[(1/a^2) - (1/b^2)](d - c)$. 779. $\frac{1}{10}$. 780. a) π ; b) $\frac{1}{2}\pi$; c) $\frac{1}{2}\pi$.
781. $\frac{1}{18}(3\sqrt{3} + 2\pi)$. 759. $\frac{1}{2}$.

C1. 3. Substituční metoda pro dvojiny integrál

745. $\frac{27}{2}$. 746. $\frac{1}{280}$. 747. $\frac{1}{60}$. 748. $\frac{15}{15}$. 749. $\frac{3}{2}$. 750. $4\sqrt{3} - \frac{3}{3}$. 751. $\frac{45}{4}a^2$. 752.
753. $-\frac{243}{4\pi}$. 754. $-\frac{40}{3}$. 755. $\frac{9}{4}$. 756. 1 . 757. $\frac{3}{2}(7 + e^{-8})$. 758.
- $\frac{1}{2}\ln 2 - 1 + \frac{1}{4}\pi$. 759. $\frac{1}{2}$.

Čl. 5. Trojný integrál na intervalu a na měřitelné množině

859. a) $\int_{-1}^1 \left(\int_{-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} \left(\int_0^{3\sqrt{1-x^2-\frac{1}{4}y^2}} f(x, y, z) dz \right) dy \right) dx;$
 b) $\int_0^3 \left(\int_{-2\sqrt{1-\frac{1}{9}z^2}}^{2\sqrt{1-\frac{1}{9}z^2}} \left(\int_{-\sqrt{1-\frac{1}{4}y^2-\frac{1}{9}z^2}}^{\sqrt{1-\frac{1}{4}y^2-\frac{1}{9}z^2}} f(x, y, z) dx \right) dy \right) dz.$

860. a) $\int_{-\sqrt{2}}^{\sqrt{2}} \left(\int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \left(\int_{-1}^{1-x^2-y^2} f(x, y, z) dz \right) dy \right) dx;$
 b) $\int_{-1}^1 \left(\int_{-\sqrt{1-z}}^{\sqrt{1-z}} \left(\int_{-\sqrt{1-y^2-z}}^{\sqrt{1-y^2-z}} f(x, y, z) dx \right) dy \right) dz.$

861. a) $\int_{-4}^4 \left(\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \left(\int_{\sqrt{x^2+y^2}}^4 f(x, y, z) dz \right) dy \right) dx;$
 b) $\int_0^4 \left(\int_{-z}^z \left(\int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x, y, z) dx \right) dy \right) dz.$

862. a) $\int_{-1}^0 \left(\int_0^{1+x} \left(\int_{-\sqrt{(1-z)^2-x^2}}^{\sqrt{(1-z)^2-x^2}} f(x, y, z) dy \right) dz \right) dx +$
 $+ \int_0^1 \left(\int_0^{1-x} \left(\int_{-\sqrt{(1-z)^2-x^2}}^{\sqrt{(1-z)^2-x^2}} f(x, y, z) dy \right) dz \right) dx;$
 b) $\int_0^1 \left(\int_{-(1-z)}^{1-z} \left(\int_{-\sqrt{(1-z)^2-y^2}}^{\sqrt{(1-z)^2-y^2}} f(x, y, z) dx \right) dy \right) dz.$

863. a) $\int_0^1 \left(\int_0^x \left(\int_0^{1-x} f(x, y, z) dy \right) dz \right) dx + \int_0^1 \left(\int_x^1 \left(\int_{z-x}^{1-x} f(x, y, z) dy \right) dz \right) dx;$
 b) $\int_0^1 \left(\int_0^z \left(\int_{z-y}^{1-y} f(x, y, z) dx \right) dy \right) dz + \int_0^1 \left(\int_z^1 \left(\int_0^{1-y} f(x, y, z) dx \right) dy \right) dz.$

864. $\frac{3}{4} - 2 \ln 2.$ **865.** $-\frac{52}{3}(4 - \sqrt{2}).$ **866.** $12 \ln 3.$ **867.** $(e^2 - 5)(e^2 - 1)(e - 1)/(2e).$ **868.** $\frac{8}{15}(31 + 12\sqrt{2} - 27\sqrt{3}).$ **869.** $4.$ **870.** $\frac{11}{360}.$ **871.** $\frac{1}{96}.$ **872.** $\frac{2}{3}.$ **873.** $\frac{3}{2} - 2 \ln 2.$ **874.** $\frac{1}{192}\pi^4 - \frac{1}{4}\pi^2 + 2.$ **875.** $\frac{1}{32}\pi.$ **876.** $6\pi(e - 1).$ **877.** $\frac{1}{16}\ln 5 - \frac{1}{60}.$ **878.** $\frac{3}{8}\sqrt{3} - \frac{1}{6}\pi.$ **879.** $\frac{13}{4}\pi.$ **880.** $\frac{59}{480}\pi a^5.$ **881.** $\frac{1}{4}\pi.$ **882.** $\frac{5832}{7}.$ **883.** $\frac{2}{15}.$ **884.** $\frac{4}{15}\pi ab^3 c.$

Čl. 6. Substituční metoda pro trojný integrál

885. $J_g(r, \varphi, \psi) = s^2 r^2 \cos^{s-1} \varphi \sin^{s-1} \varphi \cos^{2s-1} \psi \sin^{s-1} \psi.$ **886.** $\frac{8}{5}\pi a^4.$ **887.** $\frac{844}{15}\pi.$
888. $8\pi(e^2 - 1).$ **889.** $\frac{1}{2}\pi^2.$ **890.** $\frac{1}{48}.$ **891.** $\frac{1}{16}\pi(\pi - 2)a^4.$ **892.** $\frac{21}{10}\pi a^5.$ **894.** $\frac{96}{5}\sqrt{3}\pi a^5.$
895. $\frac{1}{12}\pi abc^4.$ **896.** $\frac{1}{4}\pi^2 abc.$ **897.** $8\pi.$ **898.** $\frac{1}{192}\pi.$ **899.** $\frac{21}{8}\pi.$ **900.** $\frac{13}{32}\pi.$ **901.** $\frac{16}{3}\pi.$
902. $\frac{32}{3}\pi.$ **903.** $\frac{1}{30}(108\sqrt{3} - 97)\pi a^5.$ **904.** $\frac{35}{6}\ln 3.$

Čl. 7. Některé aplikace trojnáho integrálu

905. $\frac{3}{35}.$ **906.** $\frac{7}{12}.$ **907.** $\frac{7}{24}.$ **908.** $\frac{17}{12} - 2 \ln 2.$ **909.** $\frac{32}{3}\pi.$ **910.** $2\pi.$ **911.** $\frac{8}{5}\pi.$
912. $\frac{243}{16}\pi.$ **913.** $\frac{4}{3}\pi abc.$ **914.** $\frac{1}{96}\pi.$ **915.** $\frac{7}{6}\pi.$ **916.** $\frac{1}{3}\pi(6\sqrt{3} - 5)a^3.$ **917.** $\frac{5}{12}\pi a^3.$
918. $\frac{1}{3}\pi(7 - 4\sqrt{2}).$ **919.** $\frac{7}{12}\pi a^3.$ **920.** $\frac{1}{3}\pi a^3.$ **921.** $\frac{1}{60}\pi a^3.$ **922.** $\frac{1}{8}\sqrt{2}\pi^2 a^3.$ **923.** $\frac{64}{105}\pi a^3.$ **924.** $\frac{1}{6}a^3.$ **925.** $\frac{1}{3}(2 - \sqrt{2})\pi abc.$ **926.** $\frac{1}{4}\pi^2 abc.$ **927.** $37 : 27.$ **928.** $\frac{1}{60}a^3.$
929. $\frac{1}{90}a^3.$ **930.** $\frac{4}{35}\pi a^3.$ **931.** $\frac{1}{210}a^3.$ **932.** $\frac{49}{864}a^3.$ **933.** $\frac{1}{10}(b - a)(d - c)(q^2 - p^2).$

1038. $\int \int (x + 3y) dx dy$. 1039. $\int \int y^2 dx dy$. 1040. $-2\pi ab$. 1041. $-\frac{1}{4}(e^{\pi} + 1)$.
1042. $\frac{1}{12}\pi \ln 2$. 1043. $-\frac{3}{2}\pi a^2$. 1044. $-\frac{1}{2}\pi a^2$. 1047. a) $\frac{1}{128}\pi a^2$; b) $\frac{1}{5}a^2$; c) $\frac{35}{2}a^2$. 1048. $(n+1)(n+2)\pi r^2$. 1049. $(n-1)(n-2)\pi r^2$. 1050. $\frac{3}{2}a^2$. 1051. $(\pi a, \frac{5}{6}a)$. 1052. $(\frac{5}{6}a, \frac{1}{6}a)$.
1053. $\frac{84}{1}a^4$.

C1. 11. Greenova věta

1022. a) $\frac{3}{4}$; b) 0; c) $\frac{12}{5}$; d) -4; e) 4. 1023. a)-e) -1. 1024. 0. 1025. -2a. 1026. $4\pi ab$.
1027. $-\frac{3}{4}$. 1028. $-\frac{8}{3}$. 1029. $\frac{1}{15}$. 1030. $a-b$. 1031. $-\pi a^2 b$. 1032. $(\frac{1}{6} + \frac{1}{16}\sqrt{2})a^3$.
1033. $-\pi a^2$. 1034. $4a^3$. 1035. $-\frac{1}{4}\pi a^3$. 1036. 0. 1037. $\frac{2}{3}[\ln(a^2 + b^2) - \ln(a^2)]$.

C1. 10. Křivkový integrál druhého druhu

1002. $R \frac{\sin a}{a}$. 1003. $(\frac{2}{5}e^{2x} + 1)a, -\frac{1}{5}e^{2x} + 1)a$. 1004. $(\frac{5}{4}a, \frac{5}{4}a)$. 1005. $(\frac{3}{4}a, \frac{3}{4}a)$.
1006. $(2(\frac{x^2}{2} - 6), \frac{6}{a})$. 1007. $(\frac{2}{5}a, \frac{2}{5}a)$. 1008. $(\frac{4\pi}{e^{2x} - 1}, \frac{4\pi}{e^{2x} - 1}, \frac{2\pi}{a}, \frac{2\pi}{b})$.
1009. $I_x = \frac{1024}{1664}a^3$, $I_y = \frac{1}{1664}a^3$ 1010. $I_x = I_y = \frac{3}{2}a^3$. 1011. $\frac{1}{4}\sqrt{6} - \ln(\sqrt{2} + \sqrt{3})$.
1012. πa^3 . 1013. $I_x = I_y = \frac{1}{15}\pi(3a^2 + 8\pi^2 b^2)\sqrt{a^2 + b^2}$, $I_z = 2\pi a^2 \sqrt{a^2 + b^2}$. 1014.
- $\frac{1}{2}(e^{4x} - 1)a^2 \sqrt{2a^2 + b^2}$. 1015. $I_x = I_y = 4H^4$.

C1. 9. Nejdříve aplikace křivkového integrálu prvního druhu

967. $\sqrt{5}\ln 2$. 968. $\sqrt{5}(2\sqrt{2} - 1)$. 969. $2(4 + 3\sqrt{2})$. 970. 24. 971. $\frac{1}{30}(25\sqrt{5} - 6\sqrt{3})$.
972. $\frac{1}{2}(5\sqrt{5} - 2\sqrt{2})$. 973. $2a^2$. 974. $\frac{3}{2}\sqrt{2}a^3$. 975. $4a^3$. 976. $4\pi a^3$. 977.
978. $2(e^a - 1) + \frac{1}{4}\pi ae^a$. 979. $\frac{ab}{\sqrt{1+b^2}}(1 - e^{-2ab})$. 980. $\frac{a}{\pi}$. 981.
- $\frac{ab}{\sqrt{a^2 + b^2}}$ arctg $\frac{a}{b}$. 982. $\frac{2}{3}\sqrt{2}[(1 + 2\pi^2)^{\frac{3}{2}} - 1]$. 983. $2\pi a^2$. 984. $\frac{1}{2}(1 + \pi)a^3$.
- 4a². 996. $2(2 - \sqrt{2})a^2$. 997. $\frac{3}{2}[(1 + \phi_0^2)^{\frac{3}{2}} - 1]a^2$. 998. $\frac{3}{2}ab(a^2 + ab + b^2)$. 999.
5. 990. $\sqrt{2a^2 + b^2}(e^T - e^{-T})$. 992. $\frac{1}{16}(10\sqrt{10} - 1)$. 993. $\frac{11}{12}$. 994. $4a^2$. 995.
985. $\frac{3}{2}\pi a$. 986. $[1 + \frac{1}{2}\sqrt{2}\ln(1 + \sqrt{2})]a$. 987. $8(n+1)r$. 988. $8(n-1)r$. 989.

C1. 8. Křivkový integrál prvního druhu

943. (0, 0, $\frac{3}{8}a$). 944. (1, 1, $\frac{3}{8}a$). 945. (0, 0, $\frac{5}{8}(6\sqrt{3} + 5)a$). 946. (0, 0, $\frac{3}{16}(2 + \sqrt{2})a$).
947. (0, 0, $\frac{3}{8}c$). 948. (0, 0, $\frac{3}{4}\sqrt{2}\ln(\sqrt{2} + 1) - \frac{a}{2}$). 949. $\frac{1}{4}H$. 950. $(\frac{3}{4}a, \frac{5}{6}a, \frac{7}{8}a)$. 951.
- (0, 0, $\frac{1}{16}a$). 952. $\frac{1}{60}abc(a^2 + b^2)$. 953. $\frac{1}{10}(8 - 5\sqrt{2})a^5$. 954. $\frac{1}{15}\pi abc(a^2 + b^2)$. 955.
- $\frac{225}{4}(15\pi - 26)a^5$. 956. $\frac{15}{256}\sqrt{2}a^5$. 957. $\frac{1}{2}\pi H(R_2^2 - R_4^2)$. 958. $\frac{3}{2}\pi F_4 H$. 959. a) $\frac{1}{10}\pi H$;
- b) $\frac{1}{15}\pi R^2 H(3R^2 + 2H^2)$. 960. $\frac{1}{5}a$. 961. a) $\frac{3}{2}a^5$; b) $\frac{1}{6}a^5$. 962. $I_x = I_y = \frac{1}{4}\pi^2 ab^2(4a^2 + 5b^2)$.
963. a) $\frac{4}{3}\pi H$; b) $\frac{9}{13}\pi H^6$. 964. a) $\frac{1}{15}\pi R^5 H$; b) $\frac{9}{60}\pi R^3 H(3H^2 + 10H^2)$.