





Čl. 5. Trojný integrál na intervalu a na měřitelné množině

859. a)  $\int_{-1}^1 \left( \int_{-2\sqrt{1-x^2}}^{2\sqrt{1-x^2}} \left( \int_0^{3\sqrt{1-x^2-\frac{1}{4}y^2}} f(x, y, z) dz \right) dy \right) dx;$

b)  $\int_0^3 \left( \int_{-2\sqrt{1-\frac{1}{9}z^2}}^{2\sqrt{1-\frac{1}{9}z^2}} \left( \int_{-\sqrt{1-\frac{1}{4}y^2-\frac{1}{9}z^2}}^{\sqrt{1-\frac{1}{4}y^2-\frac{1}{9}z^2}} f(x, y, z) dx \right) dy \right) dz.$

860. a)  $\int_{-\sqrt{2}}^{\sqrt{2}} \left( \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \left( \int_{-1}^{1-x^2-y^2} f(x, y, z) dz \right) dy \right) dx;$

b)  $\int_{-1}^1 \left( \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \left( \int_{-\sqrt{1-y^2-z}}^{\sqrt{1-y^2-z}} f(x, y, z) dx \right) dy \right) dz.$

861. a)  $\int_{-4}^4 \left( \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \left( \int_{\sqrt{x^2+y^2}}^4 f(x, y, z) dz \right) dy \right) dx;$

b)  $\int_0^4 \left( \int_{-z}^z \left( \int_{-\sqrt{z^2-y^2}}^{\sqrt{z^2-y^2}} f(x, y, z) dx \right) dy \right) dz.$

862. a)  $\int_{-1}^0 \left( \int_0^{1+x} \left( \int_{-\sqrt{(1-z)^2-x^2}}^{\sqrt{(1-z)^2-x^2}} f(x, y, z) dy \right) dz \right) dx +$   
 $+ \int_0^1 \left( \int_0^{1-x} \left( \int_{-\sqrt{(1-z)^2-x^2}}^{\sqrt{(1-z)^2-x^2}} f(x, y, z) dy \right) dz \right) dx;$

b)  $\int_0^1 \left( \int_{-(1-z)}^{1-z} \left( \int_{-\sqrt{(1-z)^2-y^2}}^{\sqrt{(1-z)^2-y^2}} f(x, y, z) dx \right) dy \right) dz.$

863. a)  $\int_0^1 \left( \int_0^x \left( \int_0^{1-x} f(x, y, z) dy \right) dz \right) dx + \int_0^1 \left( \int_x^1 \left( \int_{z-x}^{1-x} f(x, y, z) dy \right) dz \right) dx;$

b)  $\int_0^1 \left( \int_0^z \left( \int_{z-y}^{1-y} f(x, y, z) dx \right) dy \right) dz + \int_0^1 \left( \int_z^1 \left( \int_0^{1-y} f(x, y, z) dx \right) dy \right) dz.$

864.  $\frac{3}{4} - 2 \ln 2.$  865.  $-\frac{52}{3}(4 - \sqrt{2}).$  866.  $12 \ln 3.$  867.  $(e^2 - 5)(e^2 - 1)(e - 1)/(2e).$  868.

$\frac{8}{15}(31 + 12\sqrt{2} - 27\sqrt{3}).$  869.  $4.$  870.  $\frac{11}{360}.$  871.  $\frac{1}{96}.$  872.  $\frac{2}{3}.$  873.  $\frac{3}{2} - 2 \ln 2.$  874.

$\frac{1}{192}\pi^4 - \frac{1}{4}\pi^2 + 2.$  875.  $\frac{1}{32}\pi.$  876.  $6\pi(e - 1).$  877.  $\frac{1}{16} \ln 5 - \frac{1}{60}.$  878.  $\frac{3}{8}\sqrt{3} - \frac{1}{6}\pi.$  879.

$\frac{13}{4}\pi.$  880.  $\frac{59}{480}\pi a^5.$  881.  $\frac{1}{4}\pi.$  882.  $\frac{5832}{7}.$  883.  $\frac{2}{15}.$  884.  $\frac{4}{15}\pi ab^3c.$

Čl. 6. Substituční metoda pro trojný integrál

885.  $J_g(r, \varphi, \psi) = s^2 r^2 \cos^{s-1} \varphi \sin^{s-1} \varphi \cos^{2s-1} \psi \sin^{s-1} \psi.$  886.  $\frac{8}{5}\pi a^4.$  887.  $\frac{844}{15}\pi.$

888.  $8\pi(e^2 - 1).$  889.  $\frac{1}{2}\pi^2.$  890.  $\frac{1}{48}.$  891.  $\frac{1}{16}\pi(\pi - 2)a^4.$  892.  $\frac{21}{10}\pi a^5.$  894.  $\frac{96}{5}\sqrt{3}\pi a^5.$

895.  $\frac{1}{12}\pi abc^4.$  896.  $\frac{1}{4}\pi^2 abc.$  897.  $8\pi.$  898.  $\frac{1}{192}\pi.$  899.  $\frac{21}{8}\pi.$  900.  $\frac{13}{32}\pi.$  901.  $\frac{16}{3}\pi.$

902.  $\frac{32}{3}\pi.$  903.  $\frac{1}{30}(108\sqrt{3} - 97)\pi a^5.$  904.  $\frac{35}{6} \ln 3.$

Čl. 7. Některé aplikace trojného integrálu

905.  $\frac{3}{35}.$  906.  $\frac{7}{12}.$  907.  $\frac{7}{24}.$  908.  $\frac{17}{12} - 2 \ln 2.$  909.  $\frac{32}{3}\pi.$  910.  $2\pi.$  911.  $\frac{8}{5}\pi.$

912.  $\frac{243}{16}\pi.$  913.  $\frac{4}{3}\pi abc.$  914.  $\frac{1}{96}\pi.$  915.  $\frac{7}{6}\pi.$  916.  $\frac{1}{3}\pi(6\sqrt{3} - 5)a^3.$  917.  $\frac{5}{12}\pi a^3.$

918.  $\frac{1}{3}\pi(7 - 4\sqrt{2}).$  919.  $\frac{7}{12}\pi a^3.$  920.  $\frac{1}{3}\pi a^3.$  921.  $\frac{1}{60}\pi a^3.$  922.  $\frac{1}{8}\sqrt{2}\pi^2 a^3.$  923.

$\frac{64}{105}\pi a^3.$  924.  $\frac{1}{6}a^3.$  925.  $\frac{1}{3}(2 - \sqrt{2})\pi abc.$  926.  $\frac{1}{4}\pi^2 abc.$  927.  $37 : 27.$  928.  $\frac{1}{60}a^3.$

929.  $\frac{1}{90}a^3.$  930.  $\frac{4}{35}\pi a^3.$  931.  $\frac{1}{210}a^3.$  932.  $\frac{49}{864}a^3.$  933.  $\frac{1}{10}(b - a)(d - c)(q^2 - p^2).$



$$\begin{aligned} & \boxed{1038.} \int \int_M (x+3y) dx dy. \quad \boxed{1039.} \int \int_M y^2 dx dy. \quad \boxed{1040.} -2\pi ab. \quad \boxed{1041.} -\frac{1}{2}(e^\pi + 1). \\ & \boxed{1042.} \frac{1}{2}\pi \ln 2. \quad \boxed{1043.} -\frac{3}{2}\pi a^4. \quad \boxed{1044.} -\frac{8}{3}\pi a^2. \quad \boxed{1047.} a) \frac{1}{15}\pi a^2; b) \frac{1}{2}\pi a^2; c) \frac{3}{2}\pi a^2. \quad \boxed{1048.} \\ & \boxed{1053.} \frac{1}{84}a^4. \quad \boxed{1049.} (n+1)(n+2)\pi r^2. \quad \boxed{1050.} \frac{2}{3}a^2. \quad \boxed{1051.} (\pi a, \frac{6}{5}a). \quad \boxed{1052.} (\frac{5}{2}a, \frac{5}{2}a). \end{aligned}$$

## Cl. 11. Greenova věta

$$\begin{aligned} & \boxed{1016.} -\frac{1}{15}. \quad \boxed{1017.} \ln 6 - \frac{3}{5}. \quad \boxed{1018.} -\frac{1}{4}\pi. \quad \boxed{1019.} \frac{1}{56}. \quad \boxed{1020.} -2\pi a^2. \quad \boxed{1021.} -\frac{3}{4}. \\ & \boxed{1022.} a) \frac{3}{4}; b) 0; c) \frac{5}{2}; d) -4; e) 4. \quad \boxed{1023.} a) -e; b) -1. \quad \boxed{1024.} 0. \quad \boxed{1025.} -2\pi. \quad \boxed{1026.} 4\pi ab. \\ & \boxed{1027.} -\frac{3}{4}. \quad \boxed{1028.} -\frac{1}{8}. \quad \boxed{1029.} a - b. \quad \boxed{1030.} \frac{1}{35}. \quad \boxed{1031.} -\pi^2 a^2 b. \quad \boxed{1032.} (\frac{6}{5} + \frac{1}{16}\sqrt{2}\pi)a^3. \\ & \boxed{1033.} -\pi a^2. \quad \boxed{1034.} 4a^3. \quad \boxed{1035.} -\frac{4}{3}\pi a^3. \quad \boxed{1036.} 0. \quad \boxed{1037.} \frac{1}{2}[\ln(a^2 + b^2) - \ln(a^2)]. \end{aligned}$$

## Cl. 10. Křivkový integrál druhého druhu

$$\begin{aligned} & \boxed{1009.} I_x = \frac{315}{1024}a^3, I_y = \frac{315}{1664}a^3, I_z = \frac{3}{2}\pi(3a^2 + 8\pi^2 b^2)\sqrt{a^2 + b^2}, I_x = I_y = I_z = 2\pi a^2\sqrt{a^2 + b^2}. \quad \boxed{1010.} I_x = I_y = \frac{2}{3}a^3. \quad \boxed{1011.} \frac{1}{4}[5\sqrt{6} - \ln(\sqrt{2} + \sqrt{3})]. \\ & \boxed{1006.} \left(2\pi^2 - 6\right)\frac{\pi}{6}a, \frac{\pi}{6}a. \quad \boxed{1007.} (\frac{5}{2}a, \frac{5}{2}a). \quad \boxed{1008.} \left(\frac{4\pi}{e^{2\pi}-1}a, -\frac{4\pi}{e^{2\pi}-1}a, \frac{2\pi}{e^{2\pi}-1}b\right). \\ & \boxed{1002.} R \frac{\alpha}{\sin \alpha}. \quad \boxed{1003.} (\frac{5}{2}(e^{2\pi} + 1)a, -\frac{5}{2}(e^{2\pi} + 1)a). \quad \boxed{1004.} (\frac{5}{4}a, \frac{5}{4}a). \quad \boxed{1005.} (\frac{3}{4}a, \frac{3}{4}a). \\ & \boxed{1000.} \frac{3}{2}\pi\sqrt{a^2 + b^2}(3a^2 + 4\pi^2 b^2). \quad \boxed{1001.} \frac{3\pi}{\sqrt{2}}[(1 + 2\pi^2)^{\frac{2}{3}} - 1]. \\ & \boxed{996.} 2(2 - \sqrt{2})a^2. \quad \boxed{997.} \frac{3}{2}[(1 + \varphi_0^2)^{\frac{3}{2}} - 1]a^2. \quad \boxed{998.} \frac{3}{2}ab\frac{a^2 + ab + b^2}{a + b}. \quad \boxed{999.} \\ & \boxed{985.} \frac{2}{3}\pi a. \quad \boxed{986.} [1 + \frac{1}{2}\sqrt{2}\ln(1 + \sqrt{2})]a. \quad \boxed{987.} 8(n+1)\pi. \quad \boxed{988.} 8(n-1)\pi. \quad \boxed{989.} \\ & \boxed{990.} \sqrt{2a^2 + b^2}(e^{T_1} - e^{T_2}). \quad \boxed{992.} \frac{27}{16}(10\sqrt{10} - 1). \quad \boxed{993.} \frac{3}{11}. \quad \boxed{994.} 4a^2. \quad \boxed{995.} \end{aligned}$$

## Cl. 9. Některé aplikace křivkového integrálu prvního druhu

$$\begin{aligned} & \frac{ab}{\sqrt{a^2 + b^2}} \operatorname{arctg} \frac{a}{b}. \quad \boxed{982.} \frac{3}{2}\sqrt{2}[(1 + 2\pi^2)^{\frac{2}{3}} - 1]. \quad \boxed{983.} 2\pi a^2. \quad \boxed{984.} \frac{1}{2}(1 + \pi)a^3. \\ & \boxed{972.} \frac{3}{2}(5\sqrt{5} - 2\sqrt{2}). \quad \boxed{973.} 2a^2. \quad \boxed{974.} \frac{3}{2}\sqrt{2}a^3. \quad \boxed{975.} 4a^{\frac{3}{2}}. \quad \boxed{976.} 4\pi a^{\frac{3}{2}}. \quad \boxed{977.} \\ & \boxed{967.} \sqrt{5}\ln 2. \quad \boxed{968.} \sqrt{5}(2\sqrt{2} - 1). \quad \boxed{969.} 2(4 + 3\sqrt{2}). \quad \boxed{970.} 24. \quad \boxed{971.} \frac{1}{30}(25\sqrt{5} - 6\sqrt{3}). \end{aligned}$$

## Cl. 8. Křivkový integrál prvního druhu

$$\begin{aligned} & \boxed{963.} a) \frac{9}{4}\pi R^6; b) \frac{9}{13}\pi R^6. \quad \boxed{964.} a) \frac{1}{15}\pi R^5 H; b) \frac{90}{3}\pi R^3 H(3R^2 + 10H^2). \\ & \boxed{960.} \frac{1}{5}\pi. \quad \boxed{961.} a) \frac{3}{2}a^5; b) \frac{6}{5}a^5. \quad \boxed{962.} I_x = I_y = \frac{4}{3}\pi^2 ab^2(4a^2 + 5b^2). \\ & \boxed{952.} \frac{1}{16}(0, \frac{7}{16}). \quad \boxed{953.} \frac{1}{60}abc(a^2 + b^2). \quad \boxed{954.} \frac{1}{2}\pi abc(a^2 + b^2). \quad \boxed{955.} \\ & \boxed{947.} (0, 0, \frac{8}{3}c). \quad \boxed{948.} \left(0, 0, \frac{3\sqrt{2}\ln(\sqrt{2} + 1) - 2}{a}\right). \quad \boxed{949.} \frac{1}{4}H. \quad \boxed{950.} (\frac{4}{3}a, \frac{6}{5}a, \frac{8}{7}a). \quad \boxed{951.} \\ & \boxed{943.} (0, 0, \frac{8}{3}a). \quad \boxed{944.} (1, 1, \frac{3}{5}). \quad \boxed{945.} (0, 0, \frac{8}{5}(6\sqrt{3} + 5)a). \quad \boxed{946.} (0, 0, \frac{16}{3}(2 + \sqrt{2})a). \\ & \boxed{940.} (\frac{1}{4}a, \frac{1}{4}b, \frac{1}{4}c). \quad \boxed{941.} (\frac{9}{7}, 0, \frac{88}{7}). \quad \boxed{942.} (\frac{5}{3}a, \frac{5}{3}b, \frac{32}{9}\sqrt{ab}). \\ & \boxed{934.} \frac{8}{\pi}a^2. \quad \boxed{935.} \frac{20}{\pi}a^2. \quad \boxed{936.} \frac{2}{\pi}H. \quad \boxed{937.} \frac{2}{\pi}H. \quad \boxed{938.} \frac{2}{\pi}H. \quad \boxed{939.} \frac{2}{\pi}H. \quad \boxed{940.} \frac{2}{\pi}H. \end{aligned}$$