

Accepted Help

(An aid to MAT2 at FCE CTU)

Note: Please do not change it in any way including marginalia. It contains no explanatory, the knowledge of using is supposed.

A. Fundamental trigonometric formulae

$$\begin{aligned}\sin(x \pm y) &= \sin x \cos y \pm \cos x \sin y & (\text{as a consequence} \quad \sin 2x &= 2 \sin x \cos x) \\ \cos(x \pm y) &= \cos x \cos y \mp \sin x \sin y & (\text{as a consequence} \quad \cos 2x &= \cos^2 x - \sin^2 x) \\ \sin^2 x &= \frac{1}{2}(1 - \cos 2x) \\ \cos^2 x &= \frac{1}{2}(1 + \cos 2x)\end{aligned}$$

B. Some fundamental integrals

$$\begin{aligned}\int \frac{dx}{a^2 + x^2} &= \frac{1}{a} \operatorname{arctg} \frac{x}{a} \quad (a \neq 0) \\ \int \frac{dx}{x^2 - a^2} &= \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| \quad (a \neq 0) \\ \int \frac{dx}{\sqrt{a^2 - x^2}} &= \arcsin \frac{x}{a} \quad (a > 0) \\ \int \frac{dx}{\sqrt{x^2 + k}} &= \ln \left| x + \sqrt{x^2 + k} \right| \quad (k \neq 0)\end{aligned}$$

C. Some special integrals

$$\begin{aligned}\int R\left(x, \sqrt{\frac{ax+b}{cx+d}}\right) dx &\dots\dots\dots \text{substitution: } t = \sqrt{\frac{ax+b}{cx+d}} \\ \int R\left(x, \sqrt{a^2 - x^2}\right) dx &\dots\dots\dots \text{substitution: } x = a \sin t \text{ (or } x = a \cos t) \\ \int R\left(x, \sqrt{x^2 + a^2}\right) dx &\dots\dots\dots \text{substitution: } x = a \operatorname{tg} t \text{ (or } x = a \operatorname{cotg} t) \\ \int R(\sin x, \cos x) dx &\dots\dots \text{substitution: } \begin{cases} t = \operatorname{tg} \frac{x}{2} & \left(\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2 dt}{1+t^2} \right) \\ t = \sin x \\ t = \cos x \\ t = \operatorname{tg} x & \left(\sin^2 x = \frac{t^2}{1+t^2}, \cos^2 x = \frac{1}{1+t^2}, dx = \frac{dt}{1+t^2} \right) \end{cases}\end{aligned}$$

D. Some applications of the definite integral in geometry and physics

$$\begin{aligned}\text{Length of graph of a function} &\dots\dots\dots s = \int_a^b \sqrt{1 + [f'(x)]^2} dx \\ \text{Volume of a solid of revolution} &\dots\dots\dots V_x = \pi \int_a^b f^2(x) dx, V_y = 2\pi \int_a^b x f(x) dx \\ \text{Static moments of a curvilinear trapezoid} &\dots\dots\dots S_x = \frac{1}{2} \int_a^b f^2(x) dx, S_y = \int_a^b x f(x) dx \\ \text{Center of gravity of a curvilinear trapezoid} &\dots\dots\dots T = \left[\frac{S_y}{m}, \frac{S_x}{m} \right], \text{ where } m = \int_a^b f(x) dx\end{aligned}$$

Good Luck !