The first (regular) test MT02: Specimen 1 / 6

Question 1 (8 p.) $\displaystyle\int \frac{4x}{x^2+2x+1}\,\mathrm{d}x =$

a) $2\ln(x^2+2x+1) - \operatorname{arctg}(x+1) + C$

b) $4\ln|x+1| + \dfrac{4}{x+1} + C$

c) $2\ln|x+1| - \dfrac{4}{x+1} + C$

d) $\ln(x+1)^2 - \dfrac{2}{x+1} + C$

e) $4\ln|x+1| - 4\operatorname{arctg}(x+1) + C$

Question 2 (8 p.) If a primitive function F of $f(x) = \sin^3 x$ on the interval $(-\infty, +\infty)$ satisfies $F(\frac{1}{2}\pi) = 0$, then $F(2\pi) =$

a) $-\frac{2}{3}$

b) $\frac{2}{3}$

c) $\frac{1}{4}$

d) $-\frac{1}{4}$

e) 0

Question 3 (4 p.) A primitive function of $f(x) = x\cos x$ is given by the formula

$$F(x) = \alpha\cos x + \beta x\sin x$$

with constants α, β that satisfy:

a) $\alpha = -1$

b) $\alpha = 1$

c) $\alpha = \frac{1}{2}$

d) $\beta = -1$

e) $\beta = \frac{1}{2}$

Question 4 (4 p.) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{8+2x-x^2}} =$

a) $\arcsin \dfrac{x-1}{3} + C$
b) $2\sqrt{8+2x-x^2} + C$
c) $\arccos \dfrac{x-1}{3} + C$
d) $\sqrt{8+2x-x^2} + C$
e) $\dfrac{\arcsin(x-1)}{3} + C$

Question 5 (8 p.) The length of the graph of the function $f(x) = \frac{1}{6}(\mathrm{e}^{3x} + \mathrm{e}^{-3x})$, $x \in \langle -\frac{1}{3}, \frac{1}{3} \rangle$, is equal to

a) $3(\mathrm{e} - \mathrm{e}^{-1})$
b) $\frac{1}{3}(\mathrm{e}^2 - \mathrm{e}^{-2})$
c) $\frac{1}{3}(\mathrm{e} - \mathrm{e}^{-1})$
d) $4(\mathrm{e}^2 - \mathrm{e}^{-2})$
e) $\frac{1}{3}\mathrm{e}$

Question 6 (4 p.) The volume of the solid of revolution which is obtained by rotating the plane figure given by the inequalities $0 \leq x \leq \cos y$, $0 \leq y \leq \frac{1}{2}\pi$ around the x-axis is expressed by

a) $\pi \displaystyle\int_0^1 \cos^2 x \,\mathrm{d}x$
b) $\pi \displaystyle\int_0^{\frac{\pi}{2}} \arccos^2 x \,\mathrm{d}x$
c) $\pi \displaystyle\int_0^1 \arccos^2 x \,\mathrm{d}x$
d) $\pi \displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \,\mathrm{d}x$
e) $\pi \displaystyle\int_0^{\frac{\pi}{2}} \cos^2 y \,\mathrm{d}y$

[Correct answers: b – a – b – a – c – c]

The first (regular) test MT02: Specimen 2 / 6

Question 1 (8 p.) If a primitive function F of $f(x) = \dfrac{1}{1+\sin^2 x}$ on the interval $(-\infty, +\infty)$ satisfies $F(0) = 0$, then $F(\frac{1}{4}\pi) =$

a) $\operatorname{arctg} \dfrac{\sqrt{2}}{2}$

b) $\ln \dfrac{3}{2} - \operatorname{arctg} \dfrac{\sqrt{2}}{2}$

c) $\dfrac{\sqrt{2}}{2} - \ln \dfrac{3}{2}$

d) $\dfrac{\sqrt{2}}{2} \operatorname{arctg} \sqrt{2}$

e) $\dfrac{\sqrt{3}}{12}\pi$

Question 2 (8 p.) If a primitive function F of $f(x) = \dfrac{2x^3 - x}{x^4 - x^2 + 1}$ on the interval $(-\infty, +\infty)$ satisfies $F(1) = 0$, then $F(2) =$

a) $\frac{824}{169}$

b) $\frac{2672}{169}$

c) $2 \ln 13$

d) $\frac{3}{2}$

e) $\ln \sqrt{13}$

Question 3 (4 p.) If a primitive function F of $f(x) = (2x-3)\mathrm{e}^x$ on the interval $(-\infty, +\infty)$ satisfies $F(1) = -2\mathrm{e}$, then $F(2) =$

a) e^2

b) 0

c) $3\mathrm{e}^2 - 3\mathrm{e}$

d) $\mathrm{e} - \mathrm{e}^2$

e) $3\mathrm{e}^2 - \mathrm{e}$

Question 4 (4 p.) Using the substitution $t = \operatorname{cotg} x$ the integral $\displaystyle\int \frac{\operatorname{cotg}^2 x}{\sin^2 x}\,\mathrm{d}x =$

a) $-\dfrac{\operatorname{cotg}^3 x}{3} + C$

b) $\dfrac{\operatorname{cotg}^3 x}{3} + C$

c) $-\operatorname{cotg}^3 \dfrac{x}{3} + C$

d) $-\dfrac{\operatorname{cotg}(x^3)}{3} + C$

e) $\dfrac{\operatorname{cotg}(x^3)}{3} + C$

Question 5 (8 p.) The length of the graph of the function $f(x) = \frac{1}{2}\ln(\sin^2 x)$, $x \in \langle \frac{1}{3}\pi, \frac{1}{2}\pi \rangle$, is equal to

a) $\frac{1}{2}\ln 3$

b) $3\ln 3$

c) $\ln \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

d) $3\ln\sqrt{3}$

e) $\frac{1}{2}\ln\sqrt{3}$

Question 6 (4 p.) The volume of the solid of revolution which is obtained by rotating the plane figure bounded by curves $x = 4 - y^2$, $x = 8 - 2y^2$ around the y-axis is expressed by

a) $2\pi \displaystyle\int_0^2 (4-y^2)^2\,\mathrm{d}y$

b) $6\pi \displaystyle\int_0^2 (4-y^2)^2\,\mathrm{d}y$

c) $\pi \displaystyle\int_0^8 (8-2y^2)^2\,\mathrm{d}y - \pi \int_0^4 (4-y^2)^2\,\mathrm{d}y$

d) $\pi \displaystyle\int_{-2}^2 (4-y^2)^2\,\mathrm{d}y$

e) $3\pi \displaystyle\int_0^2 (4-y^2)^2\,\mathrm{d}y$

[Correct answers: d – e – d – a – a – b]

The first (regular) test MT02: Specimen 3 / 6

Question 1 (8 p.) Using the substitution $t = 1 - x^3$ we get $\int 6x^5 \cos(1 - x^3)\,\mathrm{d}x =$

a) $\int 2(\cos t - t\cos t)\,\mathrm{d}t$

b) $\int 2(t\cos t - \cos t)\,\mathrm{d}t$

c) $\int \frac{1}{3}(t\cos t - \cos t)\,\mathrm{d}t$

d) $\int 6t^5 \cos t\,\mathrm{d}t$

e) $\int 6t^3 \cos t\,\mathrm{d}t$

Question 2 (8 p.) $\int \frac{2\arcsin x + x}{\sqrt{1 - x^2}}\,\mathrm{d}x =$

a) $\arcsin^2 x - \sqrt{1 - x^2} + C$

b) $\arcsin^2 x + \sqrt{1 - x^2} + C$

c) $\arcsin x \cdot (\arcsin x + x) + C$

d) $2\arcsin^2 x + x\arcsin x + C$

e) $\left(\arcsin^2 x + \frac{x^2}{2}\right) \cdot \arcsin x + C$

Question 3 (4 p.) Integrating by parts we get $\int (2x - 1)\log x\,\mathrm{d}x =$

a) $(x^2 - x)\log x + \frac{2x - x^2}{2\ln 10} + C$

b) $(x^2 - x)\frac{\log x}{\ln 10} + \frac{2x - x^2}{2\ln 10} + C$

c) $(x^2 - x)\log x + x - \frac{x^2}{2} + C$

d) $(x^2 - x)\log x + \left(x - \frac{x^2}{2}\right)\ln 10 + C$

e) $(x^2 - x)\log x - (x^2 - x)\,\frac{\log x}{\ln 10} + C$

Question 4 (4 p.) If a primitive function F of $f(x) = \sin\left(\frac{\pi - 3x}{2}\right)$ on the interval $(-\infty, +\infty)$ satisfies $F(0) = 1$, then $F(-\pi) =$

a) 2
b) $\frac{1}{3}$
c) $\frac{5}{3}$
d) $\frac{4}{3}$
e) $\frac{2}{3}$

Question 5 (8 p.) The length of the graph of the function $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $x \in \langle 1, \mathrm{e}^2 \rangle$, is equal to

a) $\frac{1}{4}(\mathrm{e}^4 + 1)$
b) $\frac{1}{4}(\mathrm{e}^4 - 3)$
c) $\frac{1}{2}(\mathrm{e}^2 + 1)$
d) $\frac{1}{4}(\mathrm{e}^4 + 3)$
e) $4\mathrm{e}^2$

Question 6 (4 p.) The area of the plane figure bounded by curves $x = 1$, $y = 1$ a $y = \ln x$ is equal to

a) $\mathrm{e} - 2$
b) $2 + \mathrm{e}$
c) e
d) $3 - \mathrm{e}$
e) 1

[Correct answers: b – a – a – c – d – a]

The first (regular) test MT02: Specimen 4 / 6

Question 1 (8 p.) If a primitive function F of $f(x) = \dfrac{1}{2 - \mathrm{e}^{-x}}$ on the interval $(-\ln 2, +\infty)$ satisfies $F(0) = 0$, then $F(\ln 2) =$

a) $\ln 3$
b) $\frac{1}{2}\ln 3$
c) $-\ln \frac{3}{2}$
d) $\frac{1}{2}\ln(\ln 4 - 1)$
e) $\ln(\ln 4 - 1)$

Question 2 (8 p.) If a primitive function F of $f(x) = \dfrac{1}{2 + \sqrt{x+5}}$ on the interval $(-5, +\infty)$ satisfies $F(-1) = 0$, then $F(4) =$

a) $2 - 2\ln\frac{5}{4}$
b) $\ln\frac{5}{4}$
c) $2(1 + 2\ln\frac{5}{4})$
d) 0
e) $2(1 - 2\ln\frac{5}{4})$

Question 3 (4 p.) Integrating by parts we get

$$\int \frac{\ln(1-x)}{x^2}\,\mathrm{d}x = -\frac{1}{x}\ln(1-x) - \varphi(x),$$

where $\varphi(x) =$

a) $\displaystyle\int \frac{1}{1-x}\,\mathrm{d}x$
b) $\displaystyle\int \frac{x^3}{1-x}\,\mathrm{d}x$
c) $\displaystyle\int \frac{x}{1-x}\,\mathrm{d}x$
d) $\displaystyle\int \frac{1}{x(1-x)}\,\mathrm{d}x$
e) $\displaystyle\int \frac{1}{x^2(1-x)}\,\mathrm{d}x$

Question 4 (4 p.) If a primitive function F of $f(x) = \dfrac{x-4}{\sqrt[3]{x}}$ on the interval $(0, +\infty)$ satisfies $F(1) = 0$, then $F(8) =$

a) $\frac{3}{5}$
b) $-\frac{6}{5}$
c) $-\frac{19}{12}$
d) $\frac{14}{3}$
e) $\frac{131}{3}$

Question 5 (8 p.) The length of the graph of the function $f(x) = \frac{1}{3}\sqrt{x^3} - \sqrt{x}$, $x \in \langle 0, 9 \rangle$, is equal to

a) 12
b) 11
c) 13
d) 14
e) 15

Question 6 (4 p.) The area of the plane figure bounded by curves $y = ax^3$ and $y = 2x^2$ is equal to $\frac{1}{6}$, for the real positive parameter a which is equal to

a) $\frac{1}{2}$
b) 3
c) 1
d) 2
e) 0

$\Big[$Correct answers: b – e – d – a – a – d$\Big]$

The first (regular) test MT02: Specimen 5 / 6

Question 1 (8 p.) $\displaystyle\int x \operatorname{arctg} x \, \mathrm{d}x =$

a) $\dfrac{x^2-1}{2} \operatorname{arctg} x + \dfrac{x}{2} + C$

b) $\dfrac{x^2+1}{2} \operatorname{arctg} x - \dfrac{x}{2} + C$

c) $\dfrac{x^2}{2(x^2+1)} + \dfrac{1}{2} \operatorname{arctg} x + C$

d) $x \operatorname{arctg} x - \operatorname{arctg} x + C$

e) $\dfrac{x^2}{2} \operatorname{arctg} x - \dfrac{x}{2} + C$

Question 2 (8 p.) A primitive function of $f(x) = \dfrac{x^4 + 3x^3 - 3x^2 + 4x - 4}{x^2+1}$ is given by the formula

$$F(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta \ln(x^2+1)$$

with constants α, β, γ, δ that satisfy

a) $\delta = \frac{1}{2}$

b) $\delta = -\frac{1}{2}$

c) $\delta = -2$

d) $\beta = \frac{1}{2}$

e) $\gamma = -2$

Question 3 (4 p.) Using the substitution $t = \sin x$ the integral $\displaystyle\int \frac{\cos^3 x}{\sin x} \, \mathrm{d}x =$

a) $\displaystyle\int \left(t - \frac{1}{t}\right) \mathrm{d}t$

b) $\displaystyle\int \frac{1}{t} \, \mathrm{d}t$

c) $\displaystyle\int \left(\frac{1}{t} - t\right) \mathrm{d}t$

d) $\displaystyle\int \left(1 - t^2\right) \mathrm{d}t$

e) $\displaystyle\int \left(t + \frac{1}{t}\right) \mathrm{d}t$

Question 4 (4 p.) If a primitive function F of $f(x) = \dfrac{1}{x(1+\ln x)}$ on the interval $(\mathrm{e}^{-1}, +\infty)$ satisfies $F(1) = 2$, then $F(\mathrm{e}) =$

a) $2 + \ln 2$
b) $2\mathrm{e}$
c) $2 + \ln 2\mathrm{e}$
d) $\frac{3}{2}$
e) $2 - \mathrm{e}$

Question 5 (8 p.) The x-coordinate of the centre of gravity of the homogeneous plane figure which is bounded by the straight lines $x = 0$, $y = 6$ and a part of the curve $y = 2x^2$ within the first quadrant, is equal to

a) $\frac{3}{4}\sqrt{2}$
b) $\frac{3}{8}\sqrt{3}$
c) $\frac{5}{8}\sqrt{2}$
d) $\frac{5}{4}$
e) $\frac{3}{8}$

Question 6 (4 p.) The length of the graph of the function $f(x) = \arcsin 2x$ is expressed by

a) $2\displaystyle\int_0^{1/2} \sqrt{\frac{2-4x^2}{1-4x^2}}\,\mathrm{d}x$

b) $2\displaystyle\int_0^{1/2} \sqrt{\frac{5-4x^2}{1-4x^2}}\,\mathrm{d}x$

c) $\displaystyle\int_0^{1/2} \sqrt{\frac{5-4x^2}{1-4x^2}}\,\mathrm{d}x$

d) $\displaystyle\int_0^{1/2} \sqrt{\frac{2-4x^2}{1-4x^2}}\,\mathrm{d}x$

e) $\displaystyle\int_{-1}^{1} \sqrt{\frac{2-x^2}{1-x^2}}\,\mathrm{d}x$

[Correct answers: b – a – c – a – b – b]

The first (regular) test MT02: Specimen 6 / 6

Question 1 (8 p.) A primitive function of $f(x) = (x^2 + 5)\cos x$ is given by the formula

$$F(x) = x^2 \sin x + \beta x \cos x + \gamma \sin x,$$

with constants β, γ that satisfy

a) $\beta = 2$
b) $\gamma = -3$
c) $\beta = -2$
d) $\gamma = -2$
e) $\gamma = 2$

Question 2 (8 p.) Using the substitution $t = \operatorname{tg}\dfrac{x}{2}$ or manipulating the integrand the following integral $\displaystyle\int \frac{\cos x}{1+\cos x}\,\mathrm{d}x =$

a) $x - 2\operatorname{tg}\dfrac{x}{2} + C$
b) $\left(1 - \dfrac{1}{3}\operatorname{tg}^2\dfrac{x}{2}\right)\cdot \operatorname{tg}\dfrac{x}{2} + C$
c) $x - \operatorname{tg}\dfrac{x}{2} + C$
d) $\operatorname{tg}\dfrac{x}{2} + x + C$
e) $\ln\left(1 + \operatorname{tg}^2\dfrac{x}{2}\right) + C$

Question 3 (4 p.) $\displaystyle\int \frac{5\sqrt{x}}{x(1+\sqrt{x})}\,\mathrm{d}x =$

a) $\dfrac{5}{2}\ln(\sqrt{x}+1) + C$
b) $10\ln(\sqrt{x}+1) + C$
c) $-10\ln(x+1) + C$
d) $5\ln\dfrac{\sqrt{x}}{1+\sqrt{x}} + C$
e) $10\ln\dfrac{\sqrt{x}}{1+\sqrt{x}} + C$

Question 4 (4 p.) If a primitive function F of $f(x) = \dfrac{\cos 2x}{\cos x - \sin x}$ on the interval $(-\frac{3}{4}\pi, \frac{1}{4}\pi)$ satisfies $F(0) = 0$, then $F(\frac{1}{6}\pi) =$

a) $\frac{3}{2}(\sqrt{3} - 1)$
b) $-\frac{3}{2}(\sqrt{3} + 1)$
c) $-\frac{1}{2}(\sqrt{3} - 1)$
d) $\frac{1}{2}(3 - \sqrt{3})$
e) $-\frac{1}{2}(\sqrt{3} + 7)$

Question 5 (8 p.) The x-coordinate of the centre of gravity of the homogeneous plane figure which is bounded by the straight lines $x = \frac{1}{8}$, $y = 5$ and the curve $y = \frac{1}{8}\sqrt{x} + 5$, is equal to

a) $\frac{3}{40}$
b) $\frac{7}{40}$
c) $\frac{20}{7}$
d) $\frac{5}{7}$
e) 6

Question 6 (4 p.) The length of the curve $\sqrt{x} + \sqrt{y} = 1$ is expressed by

a) $\displaystyle\int_0^1 \sqrt{\frac{2x - \sqrt{x} + 1}{x}}\,\mathrm{d}x$
b) $\displaystyle\int_0^1 \sqrt{\frac{x - 2\sqrt{x} + 2}{x}}\,\mathrm{d}x$
c) $\displaystyle 2\int_0^1 (x - \sqrt{x} + 1)\,\mathrm{d}x$
d) $\displaystyle\int_0^1 \sqrt{\frac{2x - 2\sqrt{x} + 1}{x}}\,\mathrm{d}x$
e) $\displaystyle\int_0^1 \sqrt{1 + (1 - \sqrt{x})^4}\,\mathrm{d}x$

[Correct answers: a – c – b – d – a – d]