The second (make-up) test MT02: Specimen 1 / 6

Question 1 (8 p.) The equation of the tangent plane to the surface $z = \sqrt{y \sin x}$ at the point $\left[\frac{1}{6}\pi, 1, \frac{1}{2}\sqrt{2}\right]$ is

a) $6x + y + 2z - 1 - \sqrt{2} - \pi = 0$

b) $2\sqrt{3}x + 2y - 4\sqrt{2}z + 2 - \dfrac{\pi}{\sqrt{3}} = 0$

c) $\dfrac{6}{\pi}x - y + \sqrt{2}z - 1 = 0$

d) $\dfrac{6}{\pi}x + 2y - \sqrt{2}z - 2 = 0$

e) $6x - y - \sqrt{2}z + 2 - \pi = 0$

Question 2 (8 p.) The function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$ has at the point $\left[-\frac{5}{3}, 0\right]$

a) a saddle point

b) a non-strict local maximum

c) a non-strict local minimum

d) a strict local minimum

e) a strict local maximum

Question 3 (4 p.) The tangent line of the curve $x\mathrm{e}^y + y - 1 = 0$ at its point $[1, 0]$ has the equation

a) $x + 2y - 1 = 0$

b) $2x - y - 2 = 0$

c) $x - 2y - 1 = 0$

d) $2x + y - 2 = 0$

e) $x + y - 1 = 0$

Question 4 (4 p.) A function $x = x(y)$ is defined implicitly by the equation $y^2 - x^3 + y^2 x - 1 = 0$ and a condition $x(1) = 0$. Then $x'(1)$

a) is equal to 0
b) is equal to -1
c) is equal to $-\frac{1}{2}$
d) does not exists
e) is equal to -2

Question 5 (4 p.) The equation of the normal line to the surface given by the equation $x^2 - 2y^2 + 2z^2 = 33$ at the point $[1, 0, 4]$ is

a) $X = [1, 0, 4] + t(1, 1, 1), \quad t \in \mathbf{R}$
b) $X = [1, 0, 4] + t(1, 8, 1), \quad t \in \mathbf{R}$
c) $X = [1, 0, 4] + t(1, 4, 8), \quad t \in \mathbf{R}$
d) $X = [1, 0, 4] + t(1, 0, 8), \quad t \in \mathbf{R}$
e) $X = [1, 0, 4] + t(0, 1, 8), \quad t \in \mathbf{R}$

Question 6 (8 p.) The lengths of the sides of a rectangle change as follows: one increases from 6 m increases by 2 mm, the other decreases from 8 m by 5 mm. The length of a diagonal of the rectangle is also changed. Using the differential of the first order, the change in length of the diagonal is approximately

a) 2.8 mm
b) −4.2 mm
c) −0.5 mm
d) −2.8 mm
e) 3.3 mm

[Correct answers: b – e – a – e – d – d]

The second (make-up) test MT02: Specimen 2 / 6

Question 1 (8 p.) The normal to the curve $\sin xy - \cos \dfrac{x}{y} - 1 = 0$ at its point $\left[\frac{1}{2}\pi, 1\right]$ has the equation

a) $x + y - \frac{1}{2}\pi = 0$
b) $\pi x + 2y - 2 - \frac{1}{2}\pi^2 = 0$
c) $2x + \pi y - 2\pi = 0$
d) $2x - \pi y = 0$
e) $x + \pi y - \frac{3}{2}\pi = 0$

Question 2 (8 p.) The function $f(x, y) = x^2 + y^3 - 2xy$ has at the point $\left[\frac{2}{3}, \frac{2}{3}\right]$

a) a non-strict local minimum
b) a non-strict local maximum
c) a saddle point
d) a strict local minimum
e) a strict local maximum

Question 3 (4 p.) The equation of the normal to the surface given by $z = \arcsin xy$ at the point $\left[\frac{1}{2}, 0, 0\right]$ is

a) $X = \left[\frac{1}{2}, 0, 0\right] + t(0, 1, -2), \ \ t \in \mathbf{R}$
b) $X = \left[\frac{1}{2}, 0, 0\right] + t(2, 1, -2), \ \ t \in \mathbf{R}$
c) $X = \left[\frac{1}{2}, 0, 0\right] + t(2, -1, 2), \ \ t \in \mathbf{R}$
d) $X = \left[\frac{1}{2}, 0, 0\right] + t(2, 0, -1), \ \ t \in \mathbf{R}$
e) $X = \left[\frac{1}{2}, 0, 0\right] + t(2, 1, 0), \ \ t \in \mathbf{R}$

Question 4 (4 p.) The maximal domain of definition of the function $f(x,y) = \sqrt{(x+1)(y-1)}$ is the set

a) $\langle -1, +\infty) \times \langle 1, +\infty)$
b) $(-\infty, -1\rangle \times (-\infty, 1\rangle$
c) $\{\langle -1, +\infty) \times \langle 1, +\infty)\} \cup \{(-\infty, -1\rangle \times (-\infty, 1\rangle\}$
d) $\{\langle \frac{1}{2}, +\infty) \times (-\infty, 2\rangle\} \cup \{(-\infty, \frac{1}{2}\rangle \times \langle 2, +\infty)\}$
e) $(-\infty, \frac{1}{2}\rangle \times (-\infty, 2\rangle$

Question 5 (4 p.) The derivative of function $f(x,y) = \ln \dfrac{y}{x}$ at point $P = [1,1]$ in the direction of a vector $\boldsymbol{u} = (u_1, u_2)$, $u_2 > 0$, where $\boldsymbol{u}$ is the normal vector of the tangent line to the curve $x^2 + y^2 - 2x = 0$ at the point P, is equal to

a) $\frac{1}{3}$
b) $-\frac{1}{3}$
c) 1
d) $\frac{1}{2}$
e) $-\frac{1}{2}$

Question 6 (8 p.) Using the differential, compute the approximate increment of function $f(x,y) = \operatorname{arctg} \dfrac{y}{x}$, if x increases from 2 to 2.1 and y decreases from 3 to 2.5.

a) -3.2
b) 0.1
c) -0.1
d) 0.7
e) 2.1

[Correct answers: b – d – a – c – c – c]

The second (make-up) test MT02: Specimen 3 / 6

Question 1 (8 p.) The lengths of the sides of a cuboid change as follows: one increases from 6 m by 2 mm, the second decreases from 8 m by 4 mm, the third increases from 5 m by 3 mm The length of a body diagonal of the cuboid is also changed. Using the differential of the first order, the change in length of the body diagonal is approximately

a) 2.8 mm
b) -4.2 mm
c) 0.14 mm
d) -2.8 mm
e) 0.33 mm

Question 2 (8 p.) The function $f(x, y) = 3x^2 + 2y^2 - 3x^2y$ has at the point $[0, 0]$

a) strict local minimum
b) a saddle point
c) strict local maximum
d) a non-strict local minimum
e) a non-strict local maximum

Question 3 (4 p.) A function $y = y(x)$ is defined implicitly by the equation $x \sin y - \cos y + \cos 2y = 0$ and a condition $y(1) = \frac{1}{2}\pi$. Then $y'(1)$

a) is equal to 0
b) is equal to $-\frac{1}{2}$
c) is equal to $-\frac{1}{3}$
d) does not exists
e) is equal to -1

Question 4 (4 p.) The derivative of function $f(x,y) = \operatorname{arctg}\dfrac{y}{x}$ at point $P = [1,1]$ in the direction of a vector $\boldsymbol{u} = (u_1, u_2)$, $u_2 > 0$, where $\boldsymbol{u}$ is the normal vector of a tangent line of the curve $x^2 + y^2 - 2x = 0$ at the point P, is equal to

a) $\frac{1}{3}$
b) $-\frac{1}{3}$
c) 1
d) $\frac{1}{2}$
e) $-\frac{1}{2}$

Question 5 (4 p.) The maximal domain of definition of the function $f(x,y) = \frac{1}{\sqrt{(x+2)(y-1)}}$ is the set

a) $\langle -1, +\infty) \times \langle 2, +\infty)$
b) $(-\infty, -2\rangle \times (-\infty, 1\rangle$
c) $\{(-\infty, -2) \times (-\infty, 1)\} \cup \{(-2, \infty) \times (1, \infty)\}$
d) $\{\langle \frac{1}{2}, +\infty) \times (-\infty, 2\rangle\} \cup \{(-\infty, \frac{1}{2}\rangle \times \langle 2, +\infty)\}$
e) $(-\infty, \frac{1}{2}\rangle \times (-\infty, 2\rangle$

Question 6 (8 p.) The equation of the tangent plane to the surface given by $z = \operatorname{arctg}(x + 2y)$ at the point $\left[\frac{1}{3}, \frac{1}{3}, \frac{\pi}{4}\right]$ is

a) $6x + y + 2z - 1 - \sqrt{2} - \pi = 0$
b) $x + 2y - 2z + \dfrac{\pi}{2} - 1 = 0$
c) $\dfrac{\pi}{4}x - y + 2z - 1 = 0$
d) $\dfrac{\pi}{4}x + 2y - 2z - 2 = 0$
e) $6x - y - 2z + 2 - \pi = 0$

[Correct answers: c – a – e – d – c – b]