Final Exam	
Mathematics 1	
Name:	Date:

(1) Given function $f(x) = \operatorname{arccotg}\left(\frac{x-2}{x}\right)$.

a) Calculate the limits $\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$, $\lim\limits_{x\to 0_+} f(x)$, $\lim\limits_{x\to 0_-} f(x)$. Find all asymptotes of the graph of the function $f(x)$.
(*4 points*)

b) Find all the points where the tangent line to the graph of the function f is perpendicular to the straight line $5x - y + 3 = 0$.
(*4 points*)

c) Find the maximal intervals where f is strictly convex and where f is strictly concave. Find all points of inflection of the graph of the function $f(x)$ (if such points exist).
(*5 points*)

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(2) Denote by M the set of all pentagons whose perimeter has length 1 m. Every such pentagon is constructed as the union of a rectangle and an equilateral triangle such that they have one common side.

a) Express the area of a triangle from M as a function s of a variable a, where a is the length of the side of triangle.
(*5 points*)

b) Determine the domain of definition $D(s)$ and calculate the derivative s' of the function s.
(*2 points*)

c) Find a point $a_0 \in D(s)$ where the function s assumes a maximal value. Verify, that this is indeed a global maximum. Determine the lengths of both sides of the rectangle forming a pentagon from M which have the maximal area.
(*5 points*)

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(3) Let $\lambda \in \mathbf{R}$ is a parameter. Given vectors

$$\mathbf{u_1} = (1, -1, 4),\ \mathbf{u_2} = (1, \lambda, -6),\ \mathbf{u_3} = (6, 9, \lambda),\ \mathbf{v} = (3, 7, 2).$$

a) Determine the set M of all values of the parameter λ for which the set of vectors $\langle \mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3} \rangle$ forms the basis of the vector space $\mathbf{R}^3$.
(*5 points*)

b) Let $\lambda = 9$. Write $\mathbf{v}$ as the linear combination of vectors $\langle \mathbf{u_1}, \mathbf{u_2} \rangle$.
(*3 points*)

c) Determine all values of the parameter λ for which $\mathbf{v}$ isn't linear combination of vectors $\langle \mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3} \rangle$.
(*5 points*)

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(4) Given points $A = [1, 1, -3]$, $B = [-2, 0, 1]$, $C = [3, 4, -1]$, $D = [8, 3, 5]$.

a) Establish the relative position of the line p passing through the points A, B and the line q passing through the points C, D.
(*4 points*)

b) Compute the area of the triangle ABC.
(*4 points*)

c) Determine a point D' mirror symmetrical to the point D with respect to the plane ϱ passing through the points A, B, C.
(*4 points*)