

Test 2 MT01: specimen 1 of 3

Question 1 (4 p.) The set of all linear combinations of vectors $\mathbf{u} = (2, 1)$, $\mathbf{v} = (-4, -2)$ is formed by

- a) all nonzero vectors from $\mathbf{R}^2$
- b) all vectors from $\mathbf{R}^2$
- c) all nonzero multiples of vector $\mathbf{v}$
- d) all multiples of vector $\mathbf{u}$
- e) vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$

Question 2 (4 p.) Which vector has to be added to the collection of vectors $\langle (1, 0, -2), (2, 4, 8), (3, 4, 6) \rangle$ to get a basis of vector space $\mathbf{R}^3$?

- a) $(-2, 0, 4)$
- b) $(1, 0, 0)$
- c) $(1, 4, 10)$
- d) $(0, 5, 9)$
- e) it is not possible to get a basis

Question 3 (8 p.) For which $a \in \mathbf{R}$ the following system of linear equations has not a solution?

$$\begin{aligned}x - y + z &= 0 \\ 2x - 2y + az &= a \\ y + z &= a\end{aligned}$$

- a) $a = 1$
- b) $a = 2$
- c) $a = 3$
- d) $a = -1$
- e) it has a solution for all $a \in \mathbf{R}$

Question 4 (4 p.) Determine matrix $\mathbf{X}$ such that $2\mathbf{A}^T - 2\mathbf{X} = -4\mathbf{B}$, where

$$\mathbf{A} = \begin{pmatrix} 2, & 1 \\ -3, & 0 \\ 1, & 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1, & 2, & -4 \\ 0, & 5, & 1 \end{pmatrix}.$$

- a) $\mathbf{X} = \begin{pmatrix} 8, & 2 \\ 2, & 20 \\ -4, & 8 \end{pmatrix}$
- b) $\mathbf{X} = \begin{pmatrix} 4, & 1 \\ 1, & 10 \\ -2, & 4 \end{pmatrix}$
- c) $\mathbf{X} = \begin{pmatrix} 8, & 2, & -4 \\ 2, & 20, & 8 \end{pmatrix}$
- d) $\mathbf{X} = \begin{pmatrix} 4, & 1, & -7 \\ 1, & 10, & 4 \end{pmatrix}$
- e) $\mathbf{X} = \begin{pmatrix} -1, & 5, & -5 \\ -1, & 5, & -1 \end{pmatrix}$

[Correct answers: d - e - b - d]

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Otázka 1 (4 b.) Vector $(a, 3)$ is a linear combination of vectors $(-2, 4)$, $(1, -2)$ if and only if

- a) $a = 3$ b) $a = -3$ c) $a = -\frac{3}{2}$ d) $a = -\frac{2}{3}$ e) $a \in \mathbf{R}$

Question 2 (4 p.) Determine all $a \in \mathbf{R}$, for which the following system of equations has the only solution

$$\begin{aligned}x + ay &= 1 \\ 2x - y &= 2\end{aligned}$$

- a) for all $a \in \mathbf{R}$
b) $a = 1$
c) $a = -2$
d) $a \neq 3$
e) $a \neq -\frac{1}{2}$

Question 3 (8 p.) Given the following system of linear equations.

$$\begin{aligned}x - 2y &= 2a \\ 2x + 5y &= 1\end{aligned}$$

A necessary and sufficient condition for $a \in \mathbf{R}$ to be $x \leq 0$ and $y > 0$ is

- a) $a \in (1, 5)$ b) $a \in \langle \frac{3}{4}, \infty \rangle$
c) $a \in (\frac{1}{2}, 10)$ d) $a \in (-\infty, -\frac{1}{5})$
e) for none $a \in \mathbf{R}$

Question 4 (4 p.) Determine all $a \in \mathbf{R}$ for which the dimension of the linear hull of vectors $\langle (-3, a), (1, 1), (3, a) \rangle$ is equal to two.

- a) for none a b) for $a \in \mathbf{R}$
c) for $a = 0$ d) for $a = 3$
e) for $a = -3$

[Correct answers: c – e – d – b]

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Question 1 (4 p.) Determine all $a \in \mathbf{R}$ for which the following system of linear equations has no solutions.

$$\begin{aligned}x + 4y &= 2 \\ 3x + ay &= 1\end{aligned}$$

- a) $a = 1$
- b) $a = 4$
- c) $a = 12$
- d) $a \neq 1$
- e) the system has a solution for all $a \in \mathbf{R}$

Question 2 (4 p.) Evaluate $-3\mathbf{A}^2 + 2\mathbf{A} - \mathbf{E}$, where $\mathbf{A} = \begin{pmatrix} 3, & 1 \\ -2, & 1 \end{pmatrix}$.

- a) $\begin{pmatrix} 26, & 14 \\ -28, & -2 \end{pmatrix}$
- b) $\begin{pmatrix} -22, & -1 \\ -16, & -2 \end{pmatrix}$
- c) $\begin{pmatrix} 0, & -2 \\ 4, & 4 \end{pmatrix}$
- d) $\begin{pmatrix} 28, & 14 \\ -28, & 0 \end{pmatrix}$
- e) $\begin{pmatrix} -16, & -10 \\ 20, & 4 \end{pmatrix}$

Question 3 (8 p.) A basis of the linear hull of vectors $\langle (-1, -3, -7), (0, 1, 4) \rangle$ is formed by

- a) vector $(1, 3, 7)$
- b) vectors $(2, 6, 14), (1, 4, 11)$
- c) vectors $(1, 3, 7), (0, 0, 1)$
- d) vectors $(-1, -3, -7), (0, 1, 8)$
- e) vectors $(1, 3, 7), (0, 2, 8), (0, 0, 1)$

Otázka 4 (4 b.) The inverse matrix to the matrix $\begin{pmatrix} 2, & 3, & 1 \\ 0, & a, & 2 \\ 2, & -1, & -1 \end{pmatrix}$, $a \in \mathbf{R}$, exists for

- a) $a \neq 0$
- b) $a \neq 1$
- c) $a \neq 4$
- d) $a \neq 5$
- e) none $a \in \mathbf{R}$

[Correct answers: c – e – b – c]