Exam— MAT3

Name:	Date:
Lecturer:	Signature:

1(7p.)	
2(7p.)	
3(7p.)	
4(12p.)	
Z	
$\sum$	

1. Compute

$$\int_M \sqrt{\frac{4}{4-x^2-y^2}}\,\mathrm{d}A.$$

over the set

$$M = \{(x,y) : x^2 + y^2 < 2y\}.$$

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2. Compute

$$\int_M z\,\mathrm{d}V$$

over the set

$$M = \{(x,y,z) : 2 \leqslant x^2 + y^2 \leqslant 4,\ 0 < z < \sqrt{x^2+y^2}\}.$$

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3. A positive oriented curve C is the union of the segment with endpoints $(0,0)$ and $(2,0)$ and the halfcircle $x^2 + y^2 = 2x$, $y \geqslant 0$. Use Green's theorem to compute

$$\int_C y^2\,\mathrm{d}x + x^2\,\mathrm{d}y.$$

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4. Consider the differential equation

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0.$$

1(3). Verify that the functions $y_1(x) = 2x$, $y_2(x) = x^2$ and $y_3(x) = 3x^3$ form on the interval $(0,\infty)$ the fundamental system.
2(2). Do the functions $y_1(x) = x$, $y_2(x) = x^2$ and $y_3(x) = 1$ form on the interval $(0,\infty)$ the fundamental system?
3(1). Find the fundamental system on the interval $(0,\infty)$ containing functions $y_1(x) = x$ and $y_2(x) = x - x^2$.
4(1). Is there a fundamental system on the interval $(0,\infty)$ containing functions $y_1(x) = x$ and $y_2(x) = 2x$?
5(5). Find the solution of the equation satisfying the initial condition

$$y(1) = 1,\ y'(1) = 0,\ y''(1) = 0.$$

$x = r\cos\psi\cos\phi$	$x = r\cos\phi$	$x = ar\cos\phi$
$y = r\cos\psi\sin\phi$	$y = r\sin\phi$	$y = br\sin\phi$
$z = r\sin\psi$	$z = z$	
$J = r^2\cos\psi$	$J = r$	$J = abr$