Test MT03: Sample

Question 1 (4 p.) Let $M = \{(x,y) \in \mathbb{R}^2 : 4x^2 + 9y^2 \le 1 \,\wedge\, y \le 0\}$. The double integral $\displaystyle\int_M f(x,y)\,\mathrm{d}A$, where f is an arbitrary function of two variables which is continuous on M, is equal to

a) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{3}}^{0} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$ b) $\displaystyle\int_{-\frac{1}{3}}^{\frac{1}{3}} \int_{-\frac{1}{2}\sqrt{1-9y^2}}^{0} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$

c) $\displaystyle\int_{-\frac{1}{3}}^{\frac{1}{3}} \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$ d) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{1}{3}\sqrt{1-4x^2}}^{0} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$

e) $\displaystyle\int_{0}^{\frac{1}{2}} \int_{-\frac{1}{3}\sqrt{1-4x^2}}^{0} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$

Question 2 (4 p.) The area of the region bounded by the curve $(x^2+y^2)^4 = 2y^7$ by changing to polar coordinates ($x = r\cos\varphi,\ y = r\sin\varphi$), is equal to

a) $\displaystyle 2\int_0^{\pi/2} \sin^{14}\varphi\,\mathrm{d}\varphi$ b) $\displaystyle 2\int_0^{\pi} \sin^{7}\varphi\,\mathrm{d}\varphi$

c) $\displaystyle 2\int_0^{\pi} \sin^{14}\varphi\,\mathrm{d}\varphi$ d) $\displaystyle 2\int_0^{2\pi} \sin^{14}\varphi\,\mathrm{d}\varphi$

e) $\displaystyle \int_0^{\pi} \sin^{7}\varphi\,\mathrm{d}\varphi$

Question 3 (8 p.) The volume of the cylindrical solid

$$\left\{(x,y,z) \in \mathbb{R}^3 : 0 \le x \le \tfrac{1}{2}\pi \,\wedge\, 0 \le y \le \sin x \,\wedge\, 0 \le z \le x^2 + y^2\right\}$$

is equal to the double integral

a) $\displaystyle\int_0^{\pi/2} \int_0^{\sin x} (x^2+y^2)\,\mathrm{d}y\,\mathrm{d}x$ b) $\displaystyle\int_0^{\pi/2} \int_0^{x^2+y^2} \sin x\,\mathrm{d}y\,\mathrm{d}x$

c) $\displaystyle\int_0^{\pi/2} \int_0^{1} (x^2+y^2)\,\mathrm{d}x\,\mathrm{d}y$ d) $\displaystyle\int_0^{\pi/2} \int_0^{\sin(x^2+y^2)} \mathrm{d}y\,\mathrm{d}x$

e) $\displaystyle\int_0^{\sin x} \int_0^{\pi/2} (x^2+y^2)\,\mathrm{d}y\,\mathrm{d}x$

Question 4 (8 p.) The volume of the solid

$$\left\{(x,y,z)\in\mathbb{R}^3 : 0\le z\le\sqrt{x^2+y^2}\ \wedge\ x^2+y^2\le 1\ \wedge\ y\ge 0\right\}$$

is equal to the triple integral

a) $\displaystyle\int_0^{\pi/2}\int_0^1\int_0^{\varrho}\varrho\,\mathrm{d}z\,\mathrm{d}\varrho\,\mathrm{d}\varphi$ b) $\displaystyle\int_0^{\pi/2}\int_{-1}^1\int_0^{\varrho}\varrho\,\mathrm{d}z\,\mathrm{d}\varrho\,\mathrm{d}\varphi$

c) $\displaystyle\int_0^{\pi}\int_0^1\int_0^{\varrho}\varrho\,\mathrm{d}z\,\mathrm{d}\varrho\,\mathrm{d}\varphi$ d) $\displaystyle\int_0^{\pi}\int_0^1\int_0^{\varrho^2}\varrho\,\mathrm{d}z\,\mathrm{d}\varrho\,\mathrm{d}\varphi$

e) $\displaystyle\int_0^{\pi}\int_0^1\int_0^{\varrho}\varrho^2\,\mathrm{d}z\,\mathrm{d}\varrho\,\mathrm{d}\varphi$

[Answers: d – c – a – c]